The Rise and Fall of the Barcelonnettes in Mexico and their Implications for a Theory of Entrepreneurial Diasporas

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Abstract: This paper develops a socioeconomic model to analyze the construction and ulterior collapse of an entrepreneurial diaspora. The mathematical model is motivated by some empirical facts of the Barcelonnettes diaspora, a group of French immigrants who came to Mexico in the 19th century. Moreover, it makes evident that a theoretical framework that interconnects the social and economic arenas is very helpful to improve our understanding of the dynamics of any entrepreneurial diaspora. These migration-chains with a high level of entrepreneurialism depend heavily on their underlying social governance and, consequently, the sustainability of their social norms and values is critical for explaining their survival. In the case of the Barcelonnettes, a logistic map and numerical simulations show that the dynamics of this diaspora was very fragile, since its success bred its own destruction. This conclusion can be applied to other entrepreneurial diasporas with a “communitarian spin offs system”, where ethnic firms promise the recruited personnel the sponsoring of their entrepreneurial adventures.

Keywords: entrepreneurial diasporas, ethnic entrepreneurship, business history, Mexico.

Espлendор y ocaso de los Barcelonnettes en México y sus implicaciones para una teoría de diásporas empresariales

Resumen: En este artículo se elabora un modelo socioeconómico para analizar la formación y el colapso posterior de una diáspora empresarial. El modelo matemático está motivado en hechos históricos relacionados con la diáspora Barcelonnette, conformada por un grupo de inmigrantes franceses que llegaron a México en el siglo XIX. El planteamiento aquí expuesto hace evidente que un marco teórico que interconecta las arenas social y económica es muy conveniente para mejorar el entendimiento de las diásporas empresariales en general. Estas cadenas migratorias empresariales son altamente dependientes de la gobernanza social subyacente y, consecuentemente, el sostenimiento de las normas y los valores sociales es crítico para explicar su supervivencia. A partir de un mapa logístico y simulaciones nume-

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The contemporary world is highly globalized in terms of trade, capital and migration flows. In particular, the movement of people from developing countries to developed ones is astonishing. According to the World Bank (2011), more than 215 million people live outside of their country of birth, which represents approximately 3 per cent of the world population, a little bit more than the inhabitants of Brazil—the fifth largest country—and 40 per cent more than in 1990 (The Economist, 2011). Likewise, one in 10 inhabitants in developed countries is an immigrant. The Chinese and the Indian overseas communities are the two largest in the world, where the former has between 32 and 40 million members, and the latter between 20 and 30 million (Chand and Ghorbani, 2011).

Besides the social and political consequences of people crossing countries’ borders on a temporary or permanent basis, the impact of migration flows for the world economy and local development is very high. For this reason, in recent years many scholars of a wide variety of disciplines are becoming interested in a process known in academic circles as transnational networking or entrepreneurial diasporas (Nkongolo-Bakenda and Chrysostome, 2013; Kariv et al., 2009; Salaff et al., 2003). In general, a diaspora is defined as an ethnic group composed by people who emigrated from a particular nation-state and their descendants. These individuals then settled in one or several countries on a somehow permanent basis, while preserving psychic, social and material links with their place of birth. A diaspora is said to be entrepreneurial when many of its members develop and use their business skills to set up firms in the host country, and take advantage of their ethnic background and networks at home in order to undertake transnational economic ventures.

1 Here, an ethnic group is not defined as a set of genetically related individuals, but as a “cultural bearing-unit”. For more details on this issue see Barth (1969).
Diaspora entrepreneurs are frequently found and, therefore, this phenomenon is common to most societies (Dana, 2007; Vasta, 2004). In an entrepreneurial diaspora, new ethnic firms are formed with the support of migration-chains that provide financial resources, connections, training, recruitment channels and customers. A variant of entrepreneurial diasporas known as “communitarian spin offs system” helps newcomers establish their own businesses once they have proven to be competent and loyal employees (Maeztu, 2001). The credibility of the promise made to the recruited personnel of sponsoring their entrepreneurial adventures in the future depends upon community social governance, that is, upon the set of mechanisms (social norms, beliefs, ideologies) that condition individual behavior because of their embeddedness in specific social networks and communities (Bowles and Gintis, 2000).

In the literature there is still a lack of understanding of the dynamics of entrepreneurial diasporas and the sustainability of ethnic firms. This type of dynamics is, precisely, analyzed here by reviewing the experience of a diaspora of French immigrants who came to Mexico in the 19th century from Barcelonnette and its surroundings (Chabrand, 1897). Although the history of the Barcelonnettes diaspora is only a specific case, the understanding of its rise and fall can undoubtedly contribute to build a theoretical framework that can be used for explaining other entrepreneurial diasporas, either from old days or contemporary. In the paper, an overlapping generations model for the size of the business network is built, based upon micro-foundations related to the economic and social arenas. With this model it is shown that the success of an entrepreneurial diaspora with a spin offs system can also be the cause of its demise, due to the existence of negative feedbacks between the two arenas.

The rest of this paper is structured as follows. Section I presents a literature review on the dynamics of entrepreneurial diasporas. Section II deals with a brief revision of the Barcelonnettes’ bibliography and introduces an analytic narrative to explain the rise and fall of this entrepreneurial diaspora. Formal modeling starts in section III, where wages and firms’ profitability under the “communitarian spin offs system” are derived. Section IV develops a model for the formation of social norms. Section V describes logistic growth and the interconnection between social and economic arenas. Section VI presents the solution of the model by graphical means. Section VII deals with numerical simulations that reproduce the dynamics of the Barcelonnettes diaspora. The paper ends with the conclusions.
I. Literature Review on the Dynamics of Entrepreneurial Diasporas

Most studies dealing with the dynamic process of ethnic firms analyze intergenerational mobility (Borjas, 1993; Trejo, 2003), performance in ethnic entrepreneurship dynasties—second or higher generations—(Kantor, 2012; Beckers and Blumberg, 2011), or survival dynamics of firms (Georgarakos and Tatsiramos, 2009; Wennberg et al., 2011). However, a broader view is required to comprehend the survival of a cohesive chain-migration network that allows the operation of ethnic firms. In other words, the economic benefits of transnational networks can be maintained through time only if migration flows and ethnic social governance keep providing the mechanisms for the proper functioning of entrepreneurial diasporas.

According to Bello (2007), and references cited there, different versions of spin offs systems were used by ethnic firms in Mexico at the end of the 19th century. Diaspora members in this country were not attracted by salaries, which were very low in comparison to those offered in Cuba and Argentina, or by the possibility of working as employees. Instead their decision to leave their birth place was motivated by their desire to become entrepreneurs with the patronage of previous immigrants. While the Spanish spin offs system was more prone to support employees with family links, the French spin offs system was much more meritocratic and helped the most loyal and hard-working employees to become independent entrepreneurs. These cases contrast with the Lebanese community that did not establish large business networks with long-term relationships between employees and their patrons.

For more recent periods, Den Butter et al. (2007) present empirical evidence for Netherlands of ethnic firms embedded in entrepreneurial diasporas with spin offs systems. These authors argue that these networks create resources for the members of the migration-chain. Each network member expects that other members will reciprocate in the future when they receive a favor. In particular, informal ties between employees and patrons solve monitoring and bonding problems that make the employment relationship with co-ethnics more productive. In exchange for low salaries and long working hours, an employee can obtain promotions, become a partner, or take over the firm when the owner retires. Additional contemporary evidence is presented in Tulchinsky (2008) for Jews in Canada, and in Hougaz and Betta (2008) for Italians in Australia.
As opposed to other economic phenomena, the studies of entrepreneurial diasporas usually combine the analysis of market structure and incentives with sociocultural factors rooted in their ethnic background (Chand and Ghorbani, 2011; Volery, 2007). This multidimensional perspective is explained by the fact that scholars in management, social psychology, sociology, and organizational studies are the main contributors to this line of research. On the one hand, over-confidence, innovativeness, risk-taking, tolerance of uncertainty and other entrepreneurial attitudes are, for many authors, heavily related to the individual’s culture. On the other hand, the social embeddedness of these ethnic firms is thought to be crucial for their proper functioning and, hence, cultural factors associated to the immigrants’ networks are considered to be very relevant (Licht and Siegel, 2005; Rath, 2000).

Only in recent years mainstream economists are starting to introduce culture as a possible determinant of economic exchanges (Guiso et al., 2006). Their reluctance to advocate this type of explanations is explained mainly for instrumental reasons since such hypotheses do not fit in their neoclassical framework, with homogenous and rational agents, and because of the difficulties in testing them statistically. However, with the increasing prominence of fields like behavioral and experimental economics, the use of new data bases, and the development of better econometric tools, it is now feasible to identify differences in individuals’ preferences and beliefs in terms of cultural background and, then, to show statistically which kind of impact those beliefs and preferences have on economic outcomes.2

II. The Barcelonnettes’ Entrepreneurial Diaspora in Mexico

There are several secondary sources describing the experience of the Barcelonnettes in Mexico. In particular, Gouy (1980) and Gómez-Galvarriato (2001) identify this group of entrepreneurs as an example of ethnic networking. While the former emphasizes sociological arguments to explain their business practices, the latter provides an economic interpretation of their networks by invoking concepts of Douglas North’s new-institutionalism, such as agency problems, asymmetric information and transaction

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2 For instance, recent studies on the probability of immigrants in the US to become self-employed in entrepreneurial activities indicate that cultural characteristics of the immigrants’ country of origin are important determinants (Akeg et al., 2013; Foreman-Peck and Zhou, 2013).
costs. However, none of these authors develop an integrated socioeconomic model for understanding the enormous economic success and subsequent downfall of this entrepreneurial diaspora. The two goals of this section are to present a brief historical overview of this ethnic group and an analytic narrative that offers some insights to be formalized in the model developed in the following sections.

II.1. Some Historical Facts

The Barcelonnette diaspora in Mexico started in 1818-1820 with the Arnaud brothers, who founded Las Siete Puertas (Coste, 1994; Homps-Brousse, 2002, p. 8), a dry-goods store (retail store for clothes, gifts, etc.). A decade later, they were followed by three of their former employees in the town of Jausiers, who some years later became wealthy entrepreneurs with their own dry-goods store known as the Portal de las Flores, a landmark of the community (Arnaud, 1898, pp. 13-14). Their return to France in 1845 produced a succession of demonstration effects and emigration waves in the region of Barcelonnette. The fortunes of the newly rich returning home, as well as the construction of many “Mexican villas” and mausoleums were strong magnets for a poor region that, at the end of the 19th century, had a population of approximately 17,500 inhabitants (García-Díaz, 1991, p. 11; Homps-Brousse, 2002, p. 17).

It is estimated that about 7,000 individuals from the Ubaye Valley in the low Alps, Provence, arrived in Mexico between 1818 and 1955, and that at the peak of emigration (1870-1911) around 40-50 Barcelonnettes came to Mexico annually (Portillo, 2000, pp. 6, 24; Aranéga-Mirallès, 1992 pp. 31-34). Although there is not enough information to describe their yearly economic expansion, the sparse data that is available illustrates the increasing dynamism of this community during the last half of the 19th century and first decade of the 20th century. In 1850, there were only 9 dry-goods stores owned by these immigrants in Mexico, but by 1864, there were 32 retail shops and 13 other establishments. In 1890, there were 110 commercial establishments, 30 of them were wholesale stores. And by 1910, they had 214 stores, for a total of 257 establishments, 114 of which were located in the provinces (Arnaud, 1898, pp. 31-33, 46-52; Génin, 1933, p. 429; Gouy, 1980, p. 60).

In France, the Barcelonnettes endured a harsh physical environment characterized by long and cold winters, isolation by mountains and a rocky and unfertile land (Proal and Martin-Charpenel, 1998). They were
industrious individuals and, since the 17th century, had become accustomed to living abroad for extended periods of time and traveling very long distances to sell their products (Arnaud, 1898, pp. 3, 7, 8; Gouy, 1980, p. 36). Despite their humble background they were well-educated by the French standards of that time (Gouy, 1980, p. 40), and by the 18th century all women were literate and primary education was free. They did not receive a technical education, as some French in other parts of the country did, although they did acquire sufficient skills to earn a living. In short, through several centuries, this proud and autonomous ethnic group developed the cultural traits that enhanced its propensity to cooperate, its sense of solidarity and, thus, its likelihood of survival as a community, despite the hindrances of its natural environment.

This socio-cultural background was presumably internalized in the minds of this ethnic group when its members began their long colportage to Mexico. Their identity as Barcelonnettes was reinforced while in the host country because of their cultural distinctions: endogamy and marriage at a late age (Gouy, 1980, p. 91; Arias, 1998, p. 94); isolation from Mexican society, strong nationalism and regional attachment (Proal and Martin-Charpenel, 1998, p. 22); the recruitment of labor directly from Barcelonnettes (Gouy, 1980); housing and dining practices that fostered interaction within the community (Gouy, 1980, pp. 62, 90; Gamboa, 1989, p. 37); the formation of social institutions which provided support and socialization for the colony (Gouy, 1980, pp. 82, 89); and the acceptance of prolonged harsh working conditions that developed character and tested their loyalty (Gouy, 1980; Arnaud, 1898, p. 24; Charpenel and Charpenel, 1994; Martin-Charpenel, 1994).

II.2. An Analytic Narrative of their Success and Collapse

The historiography of the Barcelonnette community asserts that certain historical accidents were important for explaining their relative success in comparison with Mexican and other ethnic networks. Some examples include the expulsion of the Spaniards after the Independence War in 1821 and the official recognition of Mexico as a nation by France in 1827, well ahead of the US and other European nations (Gouy, 1980, p. 53); the French intervention and the Maximilian empire (Gouy, 1980, pp. 55-57; Gamboa, 1989, p. 31; Arnaud, 1898, p. 45); the Secession War in the US and the consequent boom in the Mexican textile business (Gouy, 1980, p. 55); the Franco-Prussian War of 1879 and the corresponding boycott to
German wholesalers in Mexico (Proal and Martin-Charpenel, 1998, p. 25); and the competitive advantage of the French fashion and retail industry (Arnaud, 1898, p. 42; Arias, 1998, p. 93; Pérez-Siller, 1999; Gamboa, 2004). However, this paper argues that while this entrepreneurial diaspora did indeed take advantage of these opportunities, an endogenous argument based upon their social governance seems more appealing given its dynamic features.

There are six elements of Barcelonnette social governance that allow a high rate of entrepreneurialism: 

i) strong social cohesion among community members;

ii) easy acceptance of newcomers that are hailed from the Valley of Ubaye;

iii) social esteem obtained through industriousness and productivity;

iv) priority of economic success over personal affections;

v) moral inclination to honor personal commitments made to others; and

vi) propensity of successful members to foster the community’s well-being (Gouy, 1980, p. 83, 84).

These social-cultural features produce three effects that are critical for the explanation of the Barcelonnettes’ capacity to accumulate faster, and to generate more spin offs, than any other business network in Mexico at that time. The a) ostracism effect, features (i), (iii) and (v), create a codified sense of honesty plus a mechanism of social sanctioning that produces trustworthy individuals and ameliorates agency costs in economic transactions. The b) long-term horizon effect, features (iii) and (iv), make possible the acceptance of harsh living conditions for a prolonged period of time that facilitates, in turn, the accumulation process by allowing a sustained reduction of labor and monitoring costs without affecting productivity. The c) entrepreneurial aspiration effect, features (i), (v) and (vi), have to do with the credibility of the promise made by employers of providing support (credit, inventories, a network of clients, brand name recognition or even inherit the patrons’ business) to talented and hard working employees who decide to become entrepreneurs.

This socio-cultural theory is robust in so far as it is also capable of explaining the downfall of the Barcelonnettes in Mexico as a vibrant business community. With that aim, five important empirical facts must be added to elucidate the historical context at the beginning of the 20th century: 

vii) the inflow of Barcelonnettes was curtailed by the immigration laws enacted in 1913 and the constraints imposed on foreign employees hired by limited liability companies in the 1920’s (Proal and Martin-Charpenel, 1998, p. 74; Gouy, 1980, p. 101; Aranéga-Mirallès, 1992, p. 51);

eviii) an additional reduction of countrymen due to the recruitment of
youth from the Valley and the community in Mexico to assist the First World War effort (Gouy, 1980, pp. 94-96); ix) the size of the diaspora and the disparate distribution of income that reached the community at the end of the Porfirian regime (Gómez-Galvarriato, 1999, pp. 139-140; Gómez-Galvarriato, 2001, p. 25; Gamboa, 1989, p. 56); x) the diversification of the community’s business empire (department stores, textiles factories, banks and other industries —e.g., beer, processed food, gunpowder, paper) by means of large companies that required increased funding and more technically oriented personnel (Gómez-Galvarriato, 2001, pp. 43-44; García-Díaz, 1991, pp. 34, 42; Gouy, 1980, p. 91); xi) the deterioration in the profitability of many of their important investments (textile and banking) due to the Revolutionary War and the animosity created between unions and businessmen.

In attention to these facts, the collapse in this entrepreneurial diaspora is explained by three effects that are part of the social governance explanation and one effect that involves an argument not related to the “communitarian spin offs system”. The d) future generation effect, features (vii) and (viii), put the future generation of potential entrepreneurs in jeopardy since the supply of norm-abiding individuals shrinks. The e) size and steep hierarchical structure effect, feature (ix), produces the erosion of social norms and cohesion within the community as a result of their own economic success; this creates new social values that enter into conflict with the ethnic identity. The f) diversification effect, feature (x), is also conducive to the breakdown of the social contract in contrast with an entrepreneurial diaspora where investments occur mainly in the commercial sector with small and middle-sized firms; this is so, because the promise of future support becomes less credible when the ties between management and ownership weaken, and the physical investments are not easily redeployed in times of political and violent turmoil. The g) assets-destruction effect, feature (xi), indicates that the process of capital accumulation is seriously hindered with a civil war, although its impact should only be temporary if the capacity of the community’s social governance to foster entrepreneurship is not compromised.

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3 The differences between the rich “Mexicans” and the other residents in the Valley widened through time. This is illustrated by the fact that before 1870 the successful Barcelonnettes returning home improved their old houses and adopted their traditional way of life, while after that year they built luxurious mansions surrounded by large gardens; see Homps-Brousse (2002, p. 12).
Because of these four effects, especially the endogenous ones \((e\) and \(f\)), the community lost its ability to generate a large amount of entrepreneurs in the decades of the 1920’s and 1930’s, despite the fact that between 80-90 per cent of their members remained in Mexico.\(^4\) Many of them became professionals, but others continued to be successful entrepreneurs, although their business practices did not rely any longer upon their ethnic identity (Gouy, 1980, pp. 100, 106, 107). In the following sections, a mathematical model is built to describe formally the implications of effects \((a, b, c)\) and the size component of the endogenous effect \((e)\).

III. Labor Discipline and Diaspora Entrepreneurs in a “Communitarian Spin offs System”

In this section, a model is developed to explain cooperative behavior in a long-term relationship embedded in a social network. Although this mathematical model intends to elucidate the logic involved in the interaction between an employer and his employees, it is conceivable that similar conclusions can be derived when the supplier-client, lender-borrower, manager-owner relationships are the key elements under study. As the historical narrative suggested, when the Barcelonnettes started to form spin offs, commerce was their main economic activity and, hence, a model in which wages are the most important costs seems to be a good description of reality.\(^5\) However, when the business network grew and managerial activities and other input costs became important, the social cohesion of the community was still critical for making a “promise of future businesses and profitable deals” credible for loyal and efficient managers or suppliers. Nevertheless, for reasons of mathematical tractability, the parallel relationships of entrepreneurs with suppliers, managers and lenders are not analyzed in this paper.

In the labor discipline model (also known as the efficiency wage model), the positive rent obtained by employees is the result of asymmetric (or unverifiable) information and the need to offer incentives that are in line with the interest of the principal. In general, the modeling of the current behavior of workers, in terms of an expected future income, is commonly

\(^4\) The descendants are estimated in more than 60,000; see Portillo (2000, pp. 13-14). Although Pierre Audibert (1992, p. 84), the Mexican Honorary Consul in Barcelonnette, estimates that there were around 30,000 descendants at the beginning of the 90’s, and 330 last names of families living in Mexico whose background originated in the Valley.

\(^5\) Proal and Martin-Charpenel (1998, pp. 55-64) present several examples of chains of spin offs.
used in the modern literature on labor to analyze the provision of incentives in firms. The model presented here is not the exception, yet special features of the Barcelonnettes’ social governances are taken into account to analyze wage determination and firm profitability. The inclusion of similar institutions in the labor market has been considered in other circumstances (Rosenzweig and Wolpin, 1985); however, in the “communitarian spin offs system” presented here, the behavior of individuals depends not only upon financial incentives, but also upon possibilities derived from their cultural background.

**III.1. Workers’ Inter-Temporal Decision**

Say that in each Barcelonnette firm there are T employees, where competition for workers within the community is ruled out since immigration flows are enough to cover the labor demand of firms. In the host country, the only alternative for these workers is a frictionless labor market, where a fixed real wage ($w > 0$) can be obtained. This market presents a technology that does not require labor to be monitored, nor special economic or social incentives to preclude opportunistic behavior. In contrast, the other technology available requires the building of “calculative trust” where workers receive “disciplining wages” to preclude opportunistic behavior. That is, labor contracts are not enforced by law, instead, it is assumed that a working relationship is established through a reputation mechanism where workers can obtain certain future income if they behave honestly.

This first part of the model presents a one-sided prisoners dilemma because reputation considerations may deter workers from opportunistic behavior, while entrepreneurs meet their commitments for promoting loyal employees through the enforcement of a third party. Although the latter is endogenized in the following section by modeling social norm formation in specific networks, for the time being, it is assumed that the social cohesion prevailing in the community makes entrepreneurs’ commitments credible.

The norms that form part of the Barcelonnettes’ social governance require that workers forego their compensation payments when retiring or the possibility of “inheriting” the firm in case of opportunistic behavior when young. Moreover, only members of this ethnic group can work in these firms, since outsiders do not abide by the existing social norms and

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beliefs. Outsiders have the opportunity to work in firms with a technology that also requires reputation, but that does not operate under a spinoffs system. It is assumed that these two systems of “calculative trust” are managed by individuals that belong to different social networks and, hence, their social governance creates high entry barriers that are impossible to cross. Accordingly, for both types of communities, the only alternative in the case of misbehavior is to work in the frictionless labor market where social norms are more universal.

The “calculative trust” technology exhibits a fixed-coefficients production function that generates a real income of $\rho$ per-period when employees behave properly. However, the equilibrium wages that make this technology profitable vary from one community to another as a reflection of the underlying social governance. In the case of the Barcelonnettes’ firms, workers believe that they have the possibility of becoming the entrepreneur heirs, in the sense that they can become entrepreneurs in the future with the support of their patrons. This is mathematically formalized by specifying a positive probability that workers’ income flow will switch from wages to profits and capital gains. In contrast, in the case of outsiders’ firms, workers only have the incentive to receive a compensation bonus when they have performed honestly.

The inter-temporal decision of workers is framed in an overlapping generations model (OGM) so that in time $t$ the immigrant, when young, acts as a worker, while in time $t+1$, when old, acts as an entrepreneur if the patron handed the firm to him or retires and receives a compensation bonus. Notice from figure 1 that the firm can continue operations within the community at $t+1$ as long as the former patron does not sell the firm in the market, as a result of social pressure, and new immigrants are hired. At $t+1$ the new entrepreneurs can have two sources of income: operating profits and capital gains once the firm is sold; for discounting purposes these two forms of earnings are collapsed in period $t+1$.

It is important to emphasize that decisions made by workers in time $t$ are not strategic as the term is understood in the framework of game theory. Because of the individuals’ cognitive limitations and the difficulty involved in solving sophisticated problems by backward induction, the model assumes that in time $t$ no player knows his true nature at $t+1$ once he

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7 For simplicity, it is assumed that only Barcelonnettes work in the community’s firms but, obviously, this was not the case. Nonetheless, historical evidence indicates that critical positions in the administrative staff were held most of the time by members of this ethnic group.
has become an entrepreneur. *Ex ante*, workers are only aware of the social norm of “franchising”, that is, of the percentage of entrepreneurs ($\beta$) in the community that will decide at $t+1$ to keep the firm in the community and, thus, only *ex post* will each player know if he prefers to give away or sell the firm.

**III.2. Equilibrium Wages and Some Comparative Static Exercises**

In the model, workers at $t$ have two alternatives: to cooperate (work hard and be honest) or to defect (behave opportunistically). Each of these two options generates a particular income stream. If the individual cooperates, he can become an entrepreneur at time $t+1$ and obtain operating profits and capital gains. “Inheriting” a firm is a social outcome when agents still perform as workers, yet it becomes an individual issue when the agent is converted into an entrepreneur and has to decide at the end of the period whether to sell the firm or not in the market. However, this final decision is also considered a social outcome at $t$ and, hence, it is measured in probabilistic terms.

For the honest worker, his discounted expected earnings are given by the following expression:

**Figure 1. Overlapping generations, events and decisions**

[Diagram showing overlapping generations, events, and decisions]

*Source: Author’s own elaboration.*
\[ V^h_b = \omega_t + \delta \pi_t \left( \rho - \omega_{t+1}^e T - C_{t+1}^e (T-1) \right) + \delta \pi_t S_{t+1}^e + \delta(1 - \pi_t) C_{t+1}^e \]

with \[ S_{t+1}^e = \beta_{t+1}^e y_L + (1 - \beta_{t+1}^e) y_H \]

where 0 < \delta < 1 is the discounting parameter, \( \omega_t \) and \( \omega_{t+1}^e \) are the wages paid at \( t \) and \( t+1 \) respectively, \( \pi_t \) is the probability that workers have at \( t \) of becoming an entrepreneur at \( t+1 \), \( \rho \) is the gross profit obtained with \( T \) co-operative workers, \( C_{t+1}^e \) is the expected compensation bonus, \( S_{t+1}^e \) are the expected capital gains, \( y_L \) are the dividends obtained at \( t+1 \) if a worker-converted-into-entrepreneur decides to keep the firm within the community, \( y_H \) are the capital gains derived from selling the firm’s plant and equipment in the market and 0 \( \leq y_L < y_H \), \( \beta_{t+1}^e \) is the expected percentage of entrepreneurs that at \( t+1 \) give away the firm in exchange for a dividend that covers their expenses at old age.\(^8\)

From expression (1) it is clear that in time \( t \) workers make a decision with the expectation that in time \( t+1 \) there is a probability of becoming an entrepreneur. If this event materializes, the immigrant obtains a cash flow, or operating profit, net of wages paid to newly recruited workers and of compensation paid to \( (T-1) \) former workers that were not successful in inheriting the firm. The third term on the right hand side reflects the fact that operating profits are complemented with capital gains (dividends) obtained by selling (giving away) the firm to the next generation. The last term indicates that when the firm was not inherited, the worker receives a compensation bonus in old age in exchange for his good behavior when young.

When making a decision, a worker uses conjectures with regard to the future value of several variables: \( \omega_{t+1}^e, C_{t+1}^e, \beta_{t+1}^e \). The model assumes that, in an environment of uncertainty, these conjectures are made with naïve expectations such that \( \omega_{t+1}^e = \omega_t, C_{t+1}^e = C \) and \( \beta_{t+1}^e = \beta_{t-1} \). That is, the immigrant considers that once he becomes an entrepreneur the conditions observed in the spin off system will be unaltered and, consequently, the wages he has to pay will remain the same to those when he was a worker at a young age. A similar reasoning applies for \( \beta \) where the immigrant at \( t \) establishes his beliefs in terms of the information available, therefore, he is only aware of the strength of the social norm of ‘franchising’ that existed

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\(^8\) In the Barcelonnette community it was expected that its members retired and returned to France once they had accumulated between 150,000 and 200,000 francs, leaving in Mexico between 2/3 and 3/4 of their capital in the hands of the community, in exchange for dividend payments. For more details see Gouy (1980).
at \( t-1 \). Likewise, the probability of inheriting a firm at the beginning of period \( t+1 \) and giving it away at the end of the period are closely related. Thus \( \pi_t = \beta_{t-1}/T \), where the denominator implies that the larger the competition among immigrants working in a firm, the lower is the probability of becoming an entrepreneur.

On the other hand, when a worker behaves opportunistically, he obtains a large gain once and for all since his patron fires him and the worker is ostracized by the other members of the community.\(^9\) In this case, his only alternative at old age is to work in the frictionless labor market. Under these circumstances, the worker’s discounted expected income does not include the possibility of getting profits and capital gains at old age. Accordingly:

\[
V^o_b = \alpha + \delta \omega
\]

where \( \omega \) is the wage obtained in the frictionless labor market and \( \alpha \) is the windfall gain obtained by defecting, such that \( \omega < \alpha \) and \( C > \omega \).

The firm’s patron, using “calculative trust” technology, is conscious that an economic incentive has to be offered to his employees to induce proper behavior. This is achieved when the worker obtains a larger discounted income by cooperating than by defecting, that is when \( V^h_b \geq V^o_b \). In this formulation, workers are also aware that they will all receive the required wage to cooperate and, thus, they believe that none of them will free ride upon the others. Furthermore, this wage is also convenient for entrepreneurs whose objective function is to maximize current operating profits, since paying below equilibrium wages induces full defection and \( \rho = 0 \). Therefore, by using expressions (1) and (2), it is straightforward to derive the equilibrium wages (or disciplining wages) of the model.\(^10\)

**Proposition 1.** Assuming for algebraic simplicity that \( y_L = 0 \), workers in the Barcelonnette community behave honestly when they receive a disciplining wage \( \omega \geq \omega^b \) where

\(^9\) In the Barcelonnettes’ bibliography there is a great deal of evidence to support this claim. Not only were social institutions that helped to propagate information with regard to violations of social rules, but also some members who misbehaved were, on occasions, sent back to France.

\(^10\) In a traditional game theoretical framework, where strategies are rational and common knowledge is assumed, a subgame perfect equilibrium would require additional conditions. This approach is not undertaken here and, instead, more in line with the economic sociology literature, these decisions are conditioned by the individual’s social governance. Consequently, in section five it is preferred to model entrepreneurs’ decision through social norm formation.
Notice from this expression that $\omega^b_t$ is a non-linear function of the social norm $\beta_{t-1}$. This is due to the fact that immigrants play a dual role in this model, as worker at the young age and as entrepreneur at the old age. Furthermore, expected capital gains at $t$ present an ambiguous sign with respect to the value of the social norm, they are positively related because there is an increase in the probability of inheriting, but the relationship is inverse because a higher social pressure increases the probability of giving the firm away within the community once the worker has become an owner. Up until this moment, the agents are assumed homogenous and, thus, in equilibrium, no one misbehaves if the proper wages are paid.

**Proposition 2.** Equilibrium wage increases with higher windfall gains ($\alpha$) and wages in the frictionless market ($\omega$), and decreases with higher gross profits ($\rho$), capital gains ($y$) and compensation bonus ($C$).

The proof of this proposition comes from direct differentiation of expression (3). The interpretation of the results is relatively straightforward. The employer has to offer higher salaries when the incentives for misbehavior increase ($\omega, \alpha$) and the opposite holds true when there are good economic reasons for cooperating and achieving either a compensation bonus or entrepreneurial earnings ($\rho, y$) when old.

**Proposition 3.** When $\alpha < C - \omega$ and $\rho > CT$, the “calculative trust” technology allows the firm to operate with disciplining wages that are below the windfall gain ($\alpha > \omega^b$), and more patient workers (higher $\delta$) are willing to receive lower wages in equilibrium.

The proof of this proposition is presented in the Appendix. Intuitively, when the windfall gain plus wages on the frictionless market are relatively small, it is possible that the patron can offer very low wages and still encourage cooperation through the incentive of relatively large gross profits. In this scenario, the compensation bonus by itself is enough to cover the total earnings obtained by potential defectors in their entire span of life. Moreover, when immigrants are very patient, cooperation is induced by both an appealing compensation bonus, if he retires, and attractive operating profits, if he becomes an entrepreneur.
Proposition 4. Suppose that \( y < y^* \), as stated in expression (4), then the equilibrium wages are reduced when there is an increase in the percentage of Barcelonnettes that follow the social norm of keeping their firms within the community.

\[
y < \frac{(\rho - CT) - T [\alpha + \delta (\omega - C)]}{(1 - \delta)} = y^*
\]  

The proof of the proposition is presented in the Appendix. That is, under this sufficient condition, an increase in the entrepreneurs’ credibility to support loyal workers in the future, measured through \( \beta \), reduces the minimum salaries that have to be paid to induce cooperative behavior. The ambiguity of an increase in this social norm with regard to expected capital gains disappears when the value of the firm in the market is relatively small and, consequently, the important element to induce cooperation is to have higher operating profits when workers are converted into entrepreneurs.

III.3. Profitability of Barcelonnettes’ Firms

The next step is to analyze whether the social governance of the Barcelonnette community produces competitive businesses. With that aim, it is important to compare this firm with others that use similar technology, assuming that this is freely available. As argued above, the “calculative trust” technology requires workers to have a cooperative behavior in order to exhibit positive production. That is, these firms have to pay disciplining wages in order to align the interests of the workers with those of their patron.

The other firms that use this technology belong to social networks outside the Barcelonnette community. Hence, their social governance is different. In particular, these outsiders do not have social norms that promote a spin offs system where loyal and efficient workers inherit the productive assets of the community. For the sake of simplicity, it is assumed that the only incentive for outsiders to behave properly is to receive an attractive compensation when old. Likewise, any misconduct is punished with dismissal and a relatively low salary when old in the frictionless labor market.

Accordingly, the discounted income streams for honest and opportunist behavior are given, respectively, by the following expressions:
where windfall gains ($\alpha$) and naïve conjectures ($C_{t+1}^e = C$) are identical to those presented for the Barcelonnettes. Similarly, an outsider is willing to cooperate when $V^h_o \geq V^o_o$ and, thus, disciplining wages for the community of outsiders are given by $\omega^o$, where:

$$\omega \geq \alpha + \delta (\omega - C) = \omega^o$$

recalling that $C > \omega$, then, the more patient the worker is, and the patron is aware of it, the lower is the equilibrium wage offered.

Because outsiders’ and Barcelonnette’s firms have the same technology, that is, they produce identical gross profits employing $T$ cooperative workers, the relative profitability of these two types of firms depends on the social governance that prevails in their respective community and, in consequence, on the disciplining wages that make their production possible. Therefore, the competitiveness of each firm is based exclusively on the wage differential $\omega^b - \omega^o$ obtained from expressions (3) and (6).

**Proposition 5.** Barcelonnette’s firms are more profitable than outsiders’ firms ($\omega^b - \omega^o \leq 0$) when Proposition 4 holds, yet the competitiveness of Barcelonnettes’ firms decreases when the value of the social norm ($\beta$) decreases.

The proof of this proposition is presented in the Appendix. In prose, this proposition says that in the Barcelonnette community, workers are encouraged to work diligently despite the fact that their labor earnings are small in comparison with those offered in firms with similar technology, but whose social governance does not encourage a communitarian spin offs system. Likewise, the firms’ accumulation capacity in the Barcelonnette community is positively related to the strength of the social norm of “franchising”. The first result indicates that a spin offs system gives an economic advantage to those communities endowed with the adequate social governance and, thus, offers the possibility of building up a business empire in comparison with other social groups lacking the proper socio-cultural background. The second result indicates that the competitiveness of a spin offs system can be lost when there is a reduction in social cohesion. Consequently, in the following section, the social norm will be endoge-
nized in order to analyze whether empire build up presents a negative feedback mechanism that offsets the economic advantage with certain delay.

IV. The Transformation of the Social Norm

As shown in figure 1, entrepreneurs in each period have to make a decision with regard to the future of their business. Should the firm be kept within the community or should its non-human assets be sold in the market? This decision is not based exclusively on financial incentives, since their preferences are influenced by social stigmas and, in particular, by complying with a social norm that promotes a spin offs system. All $N_t$ Barcelonnettes who own firms at $t$ face the same decision problem but, in contrast to the stage when they were employees, they are not alike. Entrepreneurs are heterogeneous because of different attitudes with respect to their likings of following a proper social conduct. Therefore, using the framework adopted by Kübler (2001) to model social norms, the problem to be solved by those Barcelonnettes that in their old age inherited a firm is as follows:

$$
\text{Maximize } U_{i+1}^i(g_i) = (y_A - \gamma g_i \sigma_i) A_i
$$

where the 0-selection indicates that the social norm is followed and, consequently, the entrepreneur keeps the firm within the community without receiving anything in exchange ($y_A = 0$). The 1-selection indicates that the immigrant disobeys the norm and the firm’s non-human assets are sold at market value ($y_A = y > 0$). From the objective function, it is clear that the rejection of the norm entails a cost due to the social pressure exerted by the community. Therefore, the net benefit of social misconduct is given by $y - \gamma g_i \sigma_i$, which is lower than the pecuniary earnings ($y$). The parameter $\sigma_i$ represents the percentage of individuals that follow the social norm according to the $i$-th entrepreneur’s beliefs. That is, the higher this percentage is, the larger are the costs of social disobedience. The parameter $g_i \in [0, 1]$ measures the preference of each individual for complying with the norm and, thus, introduces heterogeneity in the behavior of entrepreneurs; $g_i = 1$ implies that the $i$-th entrepreneur has a strong conviction for proper social conduct.

Without losing generality, a uniform distribution is considered for describing the heterogeneity of preferences, that is, $g_i \sim U[0, 1]$. All commu-
nity members are aware of this fact. Under this setting, and aggregating all individual decisions, it is possible to derive mathematically the social norm of “franchising” ($\beta$), that is, the percentage of entrepreneurs that decide to keep the firm within the community in each generation.

**Proposition 6.** When entrepreneurs have different social convictions with regard to keep the firm within the community, then, the Barcelonnettes’ social norm of “franchising” is negatively related to financial incentives ($y$) and positively related to the belief that others act with their social convictions ($s$), as explicitly stated in the following expression (short term locus):

$$
\beta_s = 1 - \frac{y}{c_s}
$$

(8)

The proof of this proposition is presented in the Appendix. Expression (8) indicates that the aggregate outcome depends on shared beliefs. Accordingly, it is necessary to formulate mathematically the dynamic of how these beliefs are transformed. Therefore, assuming that there is a certain inertia in the formation of the expectations, the following difference equation explains the evolution of shared beliefs:

$$
\sigma_{s+\varepsilon} - \sigma_s = \theta \left[ v_s(\beta_s, r) - \sigma_s \right]
$$

(9)

where $s, s + \varepsilon$ are time indexes contained in the interval $[t, t + 1]$ as explained in the following paragraph, $\theta$ is a parameter that represents the speed of adjustment, and $0 \leq v_s (.) \leq 1$ is a function that measures how much the social norm is valued within the community which, in turn, depends on the percentage of individuals that follow the norm ($\beta$) and the reputation or social esteem that the community confers to norm-abiding individuals ($0 \leq r \leq 1$).

According to expression (9), when the valuation of the norm is higher than the beliefs, these are adjusted upwardly, but slowly. More precisely, it is assumed that any $\beta$ of steady state can be reached in less than half a generation, which, in the case of the model, is presumed to last an average of 7-10 years.\(^{11}\) During this period, a worker is supposed to work hard be-

\(^{11}\) According to Arnaud (1898, p. 25), Barcelonnettes were able to endure between 15 to 20 years of hard work and humble life before becoming successful and willing to go to the Valley, to proudly enjoy their quiet and lavish lives.
fore realizing his dream of inheriting a spin off, which is a reasonable length of time for the full cultural adjustment of the type considered here. Likewise, the assumption of half a generation adjustment for the social norm simplifies the mathematical analysis (as is explained in detail below). For simplicity, a linear form is assumed for the value function \( v s(.) = \beta_s + r_t \) and hence, in steady state \( (\sigma_s = \sigma_s) \), a long term relationship between the belief and the effective followers of the social norm is obtained.

\[
\sigma_s = \beta_s + r_t
\]  

**Proposition 7.** In the dynamic system formed with equations (8)-(10) there is at least one stable steady state, which could be reached in an interior equilibrium, in a lower corner equilibrium \( (\beta^* = 0) \) or in an upper corner equilibrium \( (\beta^* = 1 - (y/\gamma)) \), where the latter is produced when \( \beta_s + r_t > 1, y/\gamma < 1 \) and \( \beta_s \) is defined by the intersection of (8) and (10). Likewise, there is only one steady state at \( \beta^* = 0 \), when \( r_t < r^* \) as stated in expression (11).

\[
r_t < 2 \left[ \frac{y}{\gamma} \right]^{1/2} - 1 = r^*
\]  

The proposition is proven with the help of a graphic analysis. The convexity of the short term locus (8) in the plane \( \beta - \sigma \), and the linearity of the long term locus (10), plus its positive intercept and 45 degree slope produces five possible scenarios that are described in the Appendix. Figure 2 presents one of these scenarios with two stable steady states (A and C) and one unstable at point B when the long term locus is given by the line \( \beta + r_o \). It also illustrates the effect of a reduction in reputation \( (\Delta r < 0) \) on the dynamics of the social norm. Notice that for \( r_1 < r_0 \), there are two new interior equilibriums but, according to the difference equation (10), only the stable at C is reached if the system was previously at C. Likewise, for \( r_2 < r_1 \), where the social esteem obtained for supporting loyal employees has fallen sharply, the percentage of Barcelonnettes that follow the norm keeps decreasing until none of them obeys the norm. Accordingly, through time it is observed that \( \beta^*_0 > \beta^*_1 > \beta^*_2 = 0 \).

The reputation or social esteem granted to those that follow the social norm is not an exogenous parameter in real life, as it has been considered so far. This needs to be endogenized into the model to establish a feedback from the economic arena to the social arena. A simple approach to tackle
this issue is to consider that reputation is a function of the number of entrepreneurs in the community ($N_t = \frac{L_t}{T}$). It is realistic to assume that an increased population of entrepreneurs tends to produce more economic and social exchanges with outsiders and other colleagues that have not internalized this type of norm-abiding preferences. This, in turn, introduces the possibility that the reasons for social esteem can be modified through the passing of time and, as a result, the social esteem from conforming to the traditional norms presents an inverse relationship with the number of firms controlled by Barcelonnettes. A reduced mathematical form of this relationship is the following:

$$r_t = f(N_t) = \frac{e^q}{e^q + e^{N_t}}$$

(12)

where $q$ is a constant parameter that is connected with the carrying capacity allowed by the market demand. One of the advantages of this exponential formulation is that $0 \leq r_t \leq 1$, as required by the assumption indicated in equation (9).
Because of this negative feedback, one would expect that an entrepreneurial diaspora built through the competitive advantage of the immigrant social governance can be jeopardized with a reduction in reputation and, accordingly, in the strength of the social norm. However, the feedback occurs with certain delay and, therefore, the introduction of cycles generates the possibility of interesting dynamics.

V. The Social Embeddedness of Firms’ Growth

The immigrants’ decisions as workers in the economic arena and as entrepreneurs in the social arena, analyzed in the previous sections, have not yet been fully connected. To close the model, it is necessary to incorporate the effect that the social norm has on the expansion of the entrepreneurial diaspora which, in turn, affects ‘franchising’ through the reputation channel. The logistic map offers a good starting point to analyze the size of the system, that is, the number of firms with ‘calculative’ trust technology owned by the Barcelonnettes. This curve, by introducing negative feedback elements, precludes the possibility of explosive growth. Under this framework, the number of firms at \( t \) is the following:

\[
N_t = \beta_{t-1} N_{t-1} + \lambda_{t-1} N_{t-1} \left[ U - N_{t-1} \right]
\]  

(13)

where the first term on the right hand side indicates that in each half a generation, only a percentage of entrepreneurs prefer to keep the business within the community supporting loyal employees (social depreciation effect); the second term involves a competitiveness effect represented by parameter \( \lambda \geq 0 \), which depends on the wage differential between the two types of firms that have “calculative trust” technology; the term in parenthesis introduces a market congestion effect through a carrying capacity parameter \( U \) that constraints growth due to limitations produced by the size of market demand.

When the difference equation (13) is divided by \( N_{t-1} \), an expression for the growth of the spin offs system is derived:

\[
1 + m_t = \beta_{t-1} + \lambda_{t-1} \left[ U - N_{t-1} \right]
\]  

(14)

where \( m_t \) is the one-period rate of growth at time \( t \).

From this expression it is clear that the growth of the entrepreneurial diaspora is constrained by its own expansion. Through the social depre-
cation effect, an increase in \( N_{t-1} \) can be accompanied by a reduction in reputation and then in the social norm \((\beta)\) because more entrepreneurs decide to sell their non-human productive assets in the market. Then, the reduction in this form of social cohesion is associated with an increase in disciplining wages paid by Barcelonnette entrepreneurs, as seen in propositions 4 and 5. This, in turn, produces a fall in the firm’s profitability and accumulation capacity \((\lambda)\). Finally, a larger \( N_{t-1} \) increases the pressure in the markets where these firms sell their products because of congestion, reducing the participation for each firm and its accumulation even more. Therefore, depending on parameter values which measure the sensibility of these three effects, it is possible that after a period of time, the rate of growth \((m_t)\) becomes negative producing a fall and, perhaps, the collapse of the Barcelonnette empire, as seen with more detail below.

The logistic map per-se is capable of originating cyclical behavior (booms and bust) as it is well known in the literature of non-linear systems (Hilborn, 2000). Additionally, the value of \( \lambda \) becomes a critical conduit for describing the cycles presented in the spin offs system since equilibrium wages change with the size of the system. Thus, an accumulation function has to be added to the model:

\[
\lambda_{t-1} = \tau(\omega_i^{o} - \omega_i^{b})
\]  

(15)

with \( \lambda = \tau(0) = 0 \).

Hence, by proposition 5, when \( \beta = 0 \) the wage differential \( \omega_i^{o} - \omega_i^{b} = 0 \) and then \( \lambda = 0 \); likewise, by the same proposition \( \partial \lambda / \partial \beta > 0 \). Therefore, a weakened social cohesion is associated with a reduced accumulation rate.

V.1. The Timing of the Social and Economic Arenas

In this variant of entrepreneurial diaspora, an employee works very hard for many years in his patron’s firm before having the opportunity of “inheriting” the business. Here, it is assumed that a generation with its two stages, as a worker and as an entrepreneur, lasts approximately 20 years. Therefore, to build a business empire with around 250 firms through spin offs, as the data shows for the Barcelonnette case, could require a long period of time depending upon some of the parameters of the logistic curve. In any case, the point to make here is that the buildup takes several generations. Obviously, the model is somehow simplified by assuming...
that the network is composed of only one entrepreneurial lineage, that is, everything started with a sole adventuresome immigrant.

On the contrary, in the case of the social arena, where the social norm of “franchising” is formed, a new steady state is reached in a period of time less than the length of half a generation, represented in the model by a unit of discrete time. It is reasonable to assume than in a span of 10 years, the social cohesion of the community can be altered by the influence of economic developments observed during the previous decade. Examples of these developments are the worsening in the income distribution and the changes in the size of the immigrant population stated in section two. Consequently, the model assumes that the timing in these two arenas is quite different, so that the negative feedback of the system size \(N_{t-1}\) on empire building through the social depreciation and competitiveness effects occurs with half a generation delay, increasing the sources of cyclical behavior in the logistic map.

Therefore, when solving the logistic equation (13) for each time \(t\), it is important to have derived in advance the steady state \(\beta\) that was achieved during time \(t - 1\). For this sequential solution, it is not only important to define the steady states obtained by equating the short and long term locus of social norm formation (see proposition 8 below), it is also necessary to understand the dynamic that takes place in the social arena within each half generation so that the computer program can identify which of the multiple steady states can be reached. The non-linearity of the model and the differential timing in the two arenas precludes the possibility of obtaining an analytical solution for the state variables: \(N_t, \beta_t, \omega_t\).

**Proposition 8.** In the social arena there are two intersections between the short and long term locus \((\beta_{inf} \text{ and } \beta_{sup})\) as indicated in expression (16). Likewise, the value of the social norm one period ahead is given by the transition rules presented in expression (17).

\[
\beta_{inf} = \frac{- (1 - r) + \left[ (1 - r)^2 + 4 \left( r - \frac{y}{\gamma} \right) \right]^{\frac{1}{2}}}{-2}
\]

\[
\beta_{sup} = \frac{- (1 - r) - \left[ (1 - r)^2 + 4 \left( r - \frac{y}{\gamma} \right) \right]^{\frac{1}{2}}}{-2}
\]
where $\beta_{sup} > 0$ and $\beta_{sup} > \beta_{inf}$

(a) if $\beta_{t-1} < \beta_{inf}$ $\Rightarrow$ $\beta_t = 0$

(b) if $\beta_{t-1} > \beta_{sup}$ $\Rightarrow$ $\beta_t = \beta_{sup}$

(c) if $\beta_{inf} < \beta_{t-1} < \beta_{sup}$ $\Rightarrow$

$$\begin{cases} 
\beta_t = \beta_{sup} \text{ when } \beta_{sup} + r_t \leq 1 \\
\beta_t = 1 - \frac{y}{\gamma} \text{ otherwise}
\end{cases}$$

(17)

The proof of this proposition is presented in the Appendix. The value of $\beta_{inf}$ can be either negative or positive, in the former case, the scenario (a) is discarded since by definition the previous value of the norm cannot be lower than zero. Notice that only in scenario (c) the new steady state can be defined by an upper corner equilibrium, because there is an increase in the value of $\beta$ when moving from $t - 1$ to $t$.

V.2. The Complete “Communitarian Spin-Offs System”

A better understanding of the workings of the model is obtained by presenting the equations (and propositions) in the order in which they are sequentially solved. Given the initial condition $\beta_0$ (or lagged values of the social norm for $t > 1$), it is possible to calculate the accumulation rate derived from the equilibrium wage differential:

$$\lambda_{t-1} = \tau (\omega^o_{t-1} - \omega^b_{t-1}) = \tau \left[ \frac{\delta \beta_{t-1} \left( \frac{\rho - CT + (1 - \beta_{t-1}) y}{T} \right) - \omega^o}{1 - \delta \beta_{t-1}} \right]$$

(15’)

with

$$\omega^o = \alpha + \delta (\omega - C)$$

Then, with $\lambda_{t-1}$ and the initial conditions $N_0, \beta_0$ (or lagged values of the spin offs system size and the social norm for $t > 1$), $N_t$ can be derived using the logistic map:

$$N_t = \begin{cases} 
\beta_{t-1} N_{t-1} + \lambda_{t-1} N_{t-1} \left[ U - N_{t-1} \right] & \text{if } N_{t-1} > 0 \\
0 & \text{if } N_{t-1} = 0
\end{cases}$$

(13’)

$$V.2. \text{The Complete "Communitarian Spin-Offs System"}$$
Notice that, in this reformulation, a negative value of the spin offs system size is explicitly restricted. A negative value is mathematically possible in the cyclical behavior of the logistic map. Therefore, when market congestion is very strong, the “war” for clients leaves no firm in the spin offs system. This is only an extreme form of collapse.

The next step is to move to the social arena, and with the size of the spin offs system, calculate the reputation function:

\[ r_t = f(N_t) = \frac{e^{\varphi}}{e^{\varphi} + e^{N_t}} \] (12)

with \( \varphi = \frac{U}{\eta} \) where \( \eta \) is a proportionality constant that produces a smoothly decreasing social esteem function.

Then, it is checked whether the social norm is in fact a lower corner equilibrium (proposition 7).

\[ \text{If } r_t < 2 \left[ \frac{y}{\gamma} \right]^{\frac{1}{2}} - 1 = r^* \rightarrow \beta_t = 0 \] (11)

Otherwise, one proceeds to obtain the intersection values between the long and short term locus (first part of proposition 8, expression 16), and with those values apply the transition rules (second part of proposition 8, expression 17) to calculate the social norm for next period.

Once beta for period \( t \) has been calculated, a new round starts. It is important to recall that the model should be solved with the following restrictions in the parameter space:

- from proposition (7)
  \[ \frac{y}{\gamma} < 1 \]

- from proposition (4)
  \[ y < \frac{(\rho - CT) + T [\alpha + \delta (\omega - C)]}{(1 - \delta)} \]

- \( \rho > CT, \omega - C < 0, \alpha > \omega \) assumptions

- from proposition (3)
  \[ \alpha < C - \omega \]

- \( \lambda_{t-1}(0) = \tau(0) = 0 \) as defined in equation (15)
VI. The Solution of the Model with a Step Reputation Function

Before proceeding with the computer simulation, it is possible to obtain a graphical solution of the OGM that offers some preliminary insights. Instead of considering a social esteem function that changes in each unitary period with variations in \( N_t \) as in (12), in this section it is assumed that \( f(N_t) \) is a three-step discrete function; that is, certain thresholds have to be surpassed for a change in reputation. By making the connection between the two arenas at a slower tempo, it is feasible to visualize the orbit of the spin-offs system size with the help of a phase diagram. Using this approach the reputation function is given by the following expression.

\[
    r_t = \begin{cases} 
      r^a & \text{if } N_t \leq N^a \\
      r^b & \text{if } N^a < N_t \leq N^b \\
      r^c & \text{if } N_t > N^b 
    \end{cases} \tag{18}
\]

with \( 0 < r^c < r^b < r^a < 1 \)

Accordingly, when the size of the system increases, the reputation for the social norm changes downwardly when crossing the thresholds and, in consequence, the reductions in the steady states of beta take place after several generations (see figure 2). The effects of the variations in the social norm and in the corresponding accumulation rate can be illustrated with a phase diagram of the logistic equation. This equation is drawn in figure 3a using the growth formulation (14). Formally speaking, this line is only a set of points because of the discrete units of time. In this diagram, the horizontal line is the locus of all values of \( N_t \) where there is a fixed point \( (N_t = N_{t-1} = N^*) \).

For values of \( N_t < N^* \), the growth rate plus one is higher than one, and thus \( m > 0 \), producing a movement of \( N_{t-1} \) to the right as pointed out by the arrows on the horizontal axis. Notice that the rate of growth is positive and diminishes when moving down the vertical axis, but it becomes negative and increases in magnitude once the steady state line has been crossed. These rates of growth produce jumps of different sizes between the points, as represented by the arrows aside the logistic line. Because the endpoint of the jump before the steady state line does not necessarily coincide with \( N^* \), there is no guarantee that a direct convergence to a fixed point exists, if at all. The presence of direct or cyclical convergence to a fixed point can be more easily seen with a one-step iterated map like...
in figure 3b, whose parameter values create a cyclical movement around the steady state.

This logistic equation assumes that $\beta$ and $\lambda$ are parameters that do not vary with changes in system size; however, as indicated above, there are social depreciation and competitiveness effects that make these parameters endogenous. Through the step reputation function, successive increases in $N_{t-1}$ produce a fall in social esteem after the thresholds $N^a$ or $N^b$ are crossed which, in turn, produce a contraction in the social norm and in the rate of accumulation, because of the corresponding increase in disciplining wages (propositions 4 and 5). Consequently, the position of the logistic line in the phase diagram varies for different ranges of $N_{t-1}$. In particular, with the fall in $\beta$ and $\lambda$, both the intercept and the magnitude of the slope decrease when moving from $[0, N^a]$ to $(N^a, N^b]$ and from this latter interval to $N_{t-1} > N^b$.

In figure 4, it is assumed that $r^c$ is low enough for the corresponding intercept in the logistic line to be below the horizontal line for steady states. Notice that when the threshold $N^b$ is reached, the dynamic of the new logistic line (Logistic C) is such that there should be a contraction in size. This implies a $N_{t-1}$ that enters into the range $(N^a, N^b]$ or $[0, N^a]$ de-
pending on the magnitude of the rate of growth \( (m) \). In any case, the lower value of \( N_{t-1} \) produces a shift back to \( r^b \) or \( r^a \) and thus, to the Logistic \( B \) or Logistic \( A \) lines, respectively. Accordingly, this scenario is capable of producing a-periodic or limit cycle in the network size.

The diagram depicts a case where the system reaches \( N' > N^b \) every \( x \) periods depending upon the size of the negative growth \( (m) \) obtained when jumping to the Logistic \( C \). With \( r^c \) the rate of growth is less than zero but no less than minus one. For some parameter values, the fall in \( N_{t-1} \) can be limited. The lesson of this story is that the presence of cycles is the key element to introduce a boom and bust scenario. This cyclical behavior can even produce a complete collapse when the fall in \( r \) produces a very negative \( m \), such that the dynamic moves \( N_{t-1} \) to zero and the communitarian system disappears according to expression (13’).

**Proposition 9.** Let the “communitarian spin offs system” be described by expressions (11), (13’), (15’), (16)-(18), and propositions (4, 5 and 7),
where the steady states in the social arena are reached in half a generation and reputation is modeled through a step-function. For an initial condition \( N_0 = 1 \) and \( 0 < \beta_0 < 1 \), there are three alternative scenarios for the dynamics of the system: i) it reaches steady state with direct or cyclical convergence through diminishing growth and two abrupt changes;\(^{12}\) ii) it presents a cyclical behavior (a-periodic or limit cycle) where size is bounded from above and below by positive values; iii) there is a system collapse \( (N_t = 0) \).

While scenario (i) requires a Logistic \( C \) with an intercept above the steady state line and scenario (iii) requires a Logistic \( C \) close to the horizontal axis, the scenario (ii) is depicted graphically in figure 4. These scenarios highlight the fact that the social depreciation, competitiveness and congestion effects contribute to create cyclical dynamics. Even if the congestion effect by itself produces only a direct convergence, the two other negative feedbacks can produce large swings in the time series of size or, even more, the total collapse of the system. However, for a more precise description of the parameter values that generates each of these scenarios, it is necessary to run some numerical simulations.

**VII. The Solution of the Model Through Numerical Simulations**

When the reputation function is continuous, as in (12), the only form to study the operation of the communitarian system is through simulation. With this purpose in mind, the non-linear system of difference equations is simplified to reduce the number of parameters. Recalling that the relevant feature of equation (15’) and propositions (4 and 5) is that the rate of accumulation has a positive relationship with the social norm for a range of capital gains \( (y < y^*) \), a more parsimonious model that meets this condition is setup with the following accumulation function:

\[
\lambda = \frac{\beta^2}{\theta}
\]

where \( \theta \) is a positive constant that substitutes the parameters \( C, \alpha, \omega, \tau, \rho, \delta, T \) and whose values are chosen to calibrate the model.

\(^{12}\) It is easy to prove that a scenario of no abrupt changes is very unlikely, since for that to happen the Logistic lines would have to intersect in the threshold points which, in turn, would imply extremely specific parameter values.
Since a loyal employee is able to inherit a spin off after 10 years of hard work, it implies that 40 data points in the simulation comprise 400 hundred years. This is a very long time series since the relevant period of the Barcelonnette immigration lasted approximately 120 years starting in 1820. In the first step, the model is calibrated to obtain a cyclical behavior with initial conditions: $\beta_0 = 0.95, N_0 = 1$ and parameter values: $y = 5, \gamma = 10, \theta = 60, U = 250, \eta = 1000$. While parameter $\eta$ produces a reputation function where the value of $\beta$ falls smoothly with an increase in $N_t$, the parameter $\theta$ is selected to create an orbit with a cyclical behavior.\(^{13}\)

By varying the capital gains coefficient ($y$) between 0 and 10, it becomes evident which are the ranges of values where the size of the spin offs system exhibits a fixed state or cyclical behavior that can produce the collapse of the entrepreneurial diaspora. Although the historical relevance of the model is reduced to a span of less than 6-7 generations, a bifurcation diagram is very helpful to understand the dynamics of the system in the long term. Through this type of numerical simulation it is found that the model presents interesting dynamics for values of $y$ within

\(^{13}\) These numerical simulations are generated with the E & F Chaos Software, produced by CeNDEF at the University of Amsterdam.
the interval (3.53, 5.72). Once these boundary values are obtained, one can compare the performance of the series of size for different values of $y$ at the historically relevant period of time. For a value of $y = 5.73$, depicted in figure 5, the fall of the entrepreneurial diaspora is produced in 8 half-generations by a reduction in the value of the social norm to the lower corner equilibrium in the social arena ($\beta = 0$). As indicated in sections four and five, a larger size is accompanied by a lower social esteem which, in turn, can produce the extinction of the norm. For a value of $y = 3$, depicted in figure 6, the collapse took place in 9 half-generations due to the effect of oscillations in size, which ended up causing a severe problem of market congestion. In this scenario, the value of $\beta$ hit a corner equilibrium at $0.7 = 1 - y/\gamma$.

**Proposition 10.** In the spin offs system, described by expressions (11), (12), (13'), (16), (17) and (19), there can be a collapse within a historically relevant period of time when capital gains are relatively high or low and the system presents a cyclical behavior induced by negative feedback effects.

Intuitively, when capital gains are relatively high (but not so high that they violate $y < y^*$), this favors the pecuniary incentives and weakens the social norm. Once the network has reached a considerable size, individu-
als keep violating the norm and the business empire within the entrepreneurial diaspora falls abruptly. On the contrary, when capital gains are relatively low, the social norm gets fixed in a corner-equilibrium quite rapidly, hence the social depreciation and competitiveness effects stop their influence on the system and, consequently, the cyclical behavior is produced only by market congestion.

Therefore, the numerical simulations help to precise the range of parameter values (e.g., capital gains) where the social depreciation and competitiveness effects are more critical for the collapse of the entrepreneurial diaspora vis à vis the market congestion effect. They also make evident the fact that cyclical behavior, and thus the value of $\lambda$, is an essential feature to produce the rise and fall of the system. The carrying capacity parameter $U$ and the initial condition for $\beta$ were selected to produce the life-cycle of the spin off system in approximately 8-10 half-generations. Obviously, changes in these parameters produce a different boom-and-bust cycle, thus, the calibration used here allowed to replicate the historical facts observed in the Barcelonnette immigration, whose empire reached around 250 firms during the last decade of the Porfirian dictatorship (1870-1910). Moreover, the scenario where the social depreciation and competitiveness effects dominate seems to be more relevant historically since there is no evidence of sharp falls at the center of the diaspora’s life cycle.

**VIII. Conclusions**

The mathematical model developed in this paper is very illustrative to study the dynamic behavior of entrepreneurial diasporas with a spin offs system, that is, a system where long-term employment relationships are established in ethnic firms and patrons promote the entrepreneurial careers of their loyal employees. Entrepreneurial diasporas, in general, are very common in the historical evidence found through different cultures and periods; however, in the literature there is not a formal theoretical model to describe the dynamics that produce changes in their social governance. By quoting some stylized facts from the Barcelonnette (French) immigration that took place in Mexico during the 19th century, the paper motivates a mathematical model based upon a particular form of social governance, where loyal employees have a positive probability of receiving a spin-off from their patrons after several years of hard work.

The model combines the framework of efficiency wages with a social norm analysis for heterogeneous agents to set up the micro-foundation for
the dynamics of the spin offs system. By making the linkages between the social and economic arenas explicit, it is possible to introduce two negative feedback effects that can produce a boom and bust scenario for “communitarian spin offs systems”. The social depreciation effect is produced when the diaspora’s success diminishes social cohesion and, hence, decreases entrepreneurs’ willingness in keeping the firm in the community. The competitiveness effect is generated when the promises made to loyal employees become less credible and, hence, wages increase while the rate of accumulation in the community diminishes. These two effects exacerbate the negative feedback created by a market congestion effect that discourages the formation of new firms due to considerations related to the size of the market.

This theoretical exercise is also a good example of how to produce historical counter-facts that help validate the internal consistency of a formal hypothesis. The literature from the Barcelonettes’ demise emphasizes three empirical features: i) the revolutionary turmoil, with an unstable economy and increased labor rigidities in the textile industry; ii) the increased restriction of immigration flows imposed in the 1910’s and 1920’s, plus iii) the fatalities of World War I, which cut short the life of many potential entrepreneurs. However, the model elaborated here indicates that even if these features had not been present, it was still possible to generate a sharp fall of the entrepreneurial diaspora around the same period of time.\footnote{This theoretical argument increases the appeal of the endogeneity hypothesis, since other business groups of pre-revolutionary Mexico (e.g., the Monterrey group) managed to reform their organizations and continued with their entrepreneurial success, despite the bleak socioeconomic scenario of the 1920’s and 1930’s.} Finally, in future research it is advisable to make a comparative study of entrepreneurial diasporas. Once the stylized facts of several variants are clarified, it is possible to extend the framework developed here in various dimensions, so that the resilience of the different ethnic firms can be analyzed.

References


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Appendix

Proof of Proposition 3
Since \( \alpha < C - \omega \), then, for \( \beta_{t-1} \in [0, 1], \beta_{t-1} \alpha < \alpha < C - \omega \). Taking the two extremes of these inequalities and multiplying both sides by \( -\delta \) one gets \( \delta(\omega - C) < -\delta \beta_{t-1} \alpha \). Then adding \( \alpha \) in both sides and solving for it, the following expression is obtained:

\[
\alpha > \frac{\alpha + \delta(\omega - C)}{1 - \delta \beta_{t-1}} > \frac{\alpha + \delta (\omega-C) - \frac{\delta \beta_{t-1}}{T} \left\{ \rho - CT + (1 - \beta_{t-1})y \right\}}{[1 - \delta \beta_{t-1}]} \]

where the second inequality is valid since the added third term is always negative, recalling that \( \rho - CT > 0 \). Therefore, from expression (3) is proved that \( \alpha > \omega^b_t \).

For the second part, the derivative of (3) with respect to \( \delta \) is given by the following expression:

\[
\frac{\partial \omega^b_t}{\partial \delta} = \frac{(\omega - C) - \frac{\beta_{t-1}}{T} \left[ \rho - CT + (1 - \beta_{t-1})y \right] + \alpha \beta_{t-1}}{[1 - \delta \beta_{t-1}]^2} \]

which has a negative sign because \( \alpha < C - \omega \) and \( \rho > CT \) imply that the numerator is lower than zero.

Proof of Proposition 4
After some algebraic manipulations, the derivative of (3) with respect to \( \beta_{t-1} \) is given by the following expression:

\[
\frac{\partial \omega^b_t}{\partial \beta_{t-1}} = \frac{\delta y \left[ 2 \beta_{t-1} - \delta \beta^2_{t-1} \right] - \frac{\delta}{T} \left[ \rho - CT + y \right] + \delta \left[ \alpha + \delta(\omega - C) \right]}{[1 - \delta \beta_{t-1}]^2} \]

which has a negative sign when the numerator is less than zero. For this to happen it is required that

\[
f(\beta_{t-1}) < \frac{\rho - CT + y}{y} - \frac{T}{y} \left[ \alpha + \delta(\omega - C) \right]
\]

where \( f(\beta_{t-1}) = 2 \beta_{t-1} - \delta \beta^2_{t-1} \)

(a.1)
since $\frac{\partial f}{\partial \beta_{t-1}} = 2 \left(1 - \delta \beta_{t-1}\right) > 0$ for all $\beta_{t-1} \in [0, 1]$. This function achieves its maximum value within this interval at $\beta_{t-1} = 1$; therefore, inequality (a.1) holds for all admissible $\beta_{t-1}$ when $f(1)$ is lower than the right hand side. Consequently, $\frac{\partial w^b_t}{\partial \beta_{t-1}} < 0$ when

$$y < \frac{(\rho - C \frac{T}{T}) - T \left[\alpha + \delta (\omega - C)\right]}{(1 - \delta)} = y^*$$

**Proof of Proposition 5**

From (3), $w^b_t = \alpha + \delta (\omega - C)$ for $\beta_{t-1} = 0$, hence, from (6), $w^b_t = \omega_t$. Then, from proposition 4, disciplining wages decrease with an increase in the value of the social norm, thus, $w^b_t < \omega_t$ for $\beta_{t-1} \in [0, 1]$; accordingly, $w^b_t - \omega_t \leq 0$ for all $\beta_{t-1}$. Moreover, this proposition also implies that $\frac{\partial (\omega - \omega_t)}{\partial \beta_{t-1}} - \frac{\partial \omega_t}{\partial \beta_{t-1}} > 0$; therefore, with a lower value of the social norms the wage gap shrinks and Barcelona's firms lose competitiveness.

**Proof of Proposition 6**

When maximizing (7), there are two possible scenarios for the linear utility function (assuming shared beliefs $\rightarrow \sigma_i = \sigma$):

I) $y - \gamma g_i \sigma > 0$, hence $A_i = 1$ is the optimal selection, since for this value $U^i = (y - \gamma g_i \sigma) 1 > 0$, which is higher than $U^i = (0 - \gamma g_i \sigma) 0 = 0$ when $A_i = 0$.

II) $y - \gamma g_i \sigma \leq 0$, hence $A_i = 0$ is the optimal solution, since $U^i = (y - \gamma g_i \sigma) 1 < 0$ for $A_i = 1$, in contrast to $U^i = (0 - \gamma g_i \sigma) 0 = 0$ for $A_i = 0$.

Accordingly, from (I) the solution is $A_i = 1$ when $g_i < y/\gamma \sigma = g_c$, and from (II) the solution is $A_i = 0$ when $g_i \geq y/\gamma \sigma$. Therefore, for $g_i \sim \mathcal{U}(0,1)$ and $\beta$ defined as the percentage of those $i$ such that $A_i = 0$ →

$$\text{Prob} \left(g_i \geq g_c\right) = \int_{g_c}^{1} f(g) \, dg = 1 - g_c \quad \rightarrow \quad \beta = 1 - \frac{y}{\gamma \sigma}$$

**Proof of Proposition 7**

The five scenarios are the following: (a) two intersection points between the two locus within the unitary square, (b) no intersection between these locus in the $(\beta, \sigma)$ space, (c) a lower intersection of the two locus within the unitary square, (d) no intersection of the two locus within the unitary square and (e) an upper intersection of the two locus within the unitary square. In scenario (a), depicted in figure 7, panel a, there are three steady states for the couple $(\beta, \sigma)$, an unstable one at point $B$ and two stable ones at points $A$ and $C$, since
Figure 7. Dynamics of the social norm

Source: Author's own elaboration.
expression (9) indicates that the difference $\sigma_{st} - \sigma_{s}$ has to be negative when the short term value $\sigma_{s}$ is above the long term locus, and the opposite result holds when the short term $\sigma_{st}$ is below the long term locus. Therefore, the system produces two steady states for the social norm, a lower corner equilibrium $\beta* = 0$ at point A and an interior equilibrium $0 < \beta* < 1$ at point C.

In scenario (b), the short term curve is always above the long term locus and, hence, the difference equation (9) indicates that $\sigma_{st}$ has to decrease through time, thus, a lower corner equilibrium is reached with $\beta* = 0$ at point A. For this scenario to hold, $r$ has to be below $r*$ for all the range of $\beta$ between 0 and 1, as proved mathematically below. In scenario (c), there are three steady states, two stable ones at A and C and one unstable at B. Because $\sigma_{st}$ by definition cannot be greater than 1, the upper corner equilibrium at C is given by $\beta* = 1 - y/\gamma < 1$, which is the value of the short term locus when $\sigma_{st} = 1$, while in the lower corner equilibrium, the social norm has vanished completely ($\beta* = 0$). Notice that point C cannot intersect with the vertical line $\beta* = 1$, since the short term locus is an asymptotic curve in the upper bound of beta. In scenario (d), there are two steady states at points A and B but only the latter one is stable with an upper corner equilibrium given by $\beta* = 1 - y/\gamma$. Finally, in scenario (e), there are two steady states at A and C, but only the interior equilibrium at B is stable.

There are no more scenarios, since the short term locus cannot cross the horizontal line $\sigma = 1$ before the lower intersection between the two locus. When $\sigma = 1 \rightarrow \beta* = 1 - y/\gamma$, hence, at this point of beta, the slope of the short term locus is given by $\gamma/\gamma > 1$, while at the same point, the slope of the long term locus is one. Thus, because of the convexity of the former locus, there cannot be a lower intersection between these two locus after point $\beta*$. 

**Proof of the Condition $r < r*$ for a Unique Steady State at $\beta* = 0$**

The two locus do not intersect when

$$\sigma = \frac{y}{\gamma(1 - \beta)} > \beta + r \rightarrow r < \frac{y}{\gamma(1 - \beta)} - \beta = h(\beta)$$

(a.2)

since $h(\beta)$ is a convex function with a minimum value at $\beta_{min} = 1 - \left[\frac{y}{\gamma}\right]^{\frac{1}{2}}$, inequality (a.2) holds for all values of $\beta$ when $r < h(\beta_{min})$, then, after some algebraic manipulations this condition is restated as

$$r < 2 \left[\frac{y}{\gamma}\right]^{\frac{1}{2}} - 1 = r*.$$
Proof of Proposition 8
Equating expressions (8) and (10) one gets a second order equation for $\beta$:

$$0 = -\beta^2 + (1 - r) \beta + (r - \gamma/y)$$

where the roots ($\beta_{inf}, \beta_{sup}$) are given by (16), whose radicand is always positive, since from proposition 7 and expression (11), there is an intersection between the two locus when $r \geq r^*$. That is $(1 + r) \geq 2 (y/\gamma)^{1/2}$ implies $1 + 2r + r^2 \geq 4 (y/\gamma)$ when both sides are raised to the second power, hence, $1 - 2r + r^2 + 4r - 4 (y/\gamma) = (1 - r)^2 + 4 (r - y/\gamma) \geq 0$.

At each $t$ one can calculate the values $\beta_{inf}$ and $\beta_{sup}$, thus, there are always three alternatives. (a) $\beta_{t-1} < \beta_{inf}$ (b) $\beta_{t-1} > \beta_{sup}$ and (c) $\beta_{inf} < \beta_{t-1} < \beta_{sup}$.

Then, according to the difference equation (9) and figure (2), in case (a) beta goes down to a corner equilibrium ($\beta_{t-1} = 0$) when $\beta_{inf} > 0$; in case (b) beta goes down to the interior equilibrium ($\beta_{t} = \beta_{sup}$); in case (c) beta goes up to the interior equilibrium ($\beta_{t} = \beta_{sup}$) when this exists, otherwise [$\beta_{t} + r_{t} > 1$] it reaches an upper corner equilibrium ($\beta_{t} = 1 - y/\gamma < 1$).

Source: Author’s own elaboration.