Optimal Growth and Disinflation under Incomplete Credit Markets

Alejandro Rodríguez-Arana*

Abstract: This paper shows that when money is necessary to consume but not to invest a gradual reduction of inflation has a positive effect on output, growth or both. If there is an externality à la Romer, a gradual and permanent disinflation could be optimal from a social point of view. In that case, the consistent monetary policy would be to reduce the growth of the nominal quantity of money also gradually.

Resumen: Este artículo muestra que cuando el dinero es necesario para comprar bienes de consumo, pero no bienes de capital, una reducción gradual de la inflación tiene un efecto positivo en la producción, el crecimiento o ambos. Si el proceso productivo está sujeto a una externalidad tipo Romer, la reducción gradual y permanente de la inflación constituiría una de las alternativas de política óptimas desde el punto de vista social. En ese caso, la política monetaria congruente sería reducir gradualmente el crecimiento de la cantidad nominal de dinero.

* Universidad Iberoamericana, Prolongación Paseo de la Reforma 880, Lomas de Santa Fe, 01210 México, D.F. Tel.: 5267-4000, ext. 4935, correo electrónico: alejandro.rodriguez@uia.mx.

Introduction

Different theoretical studies find high levels of inflation affecting growth in a negative way. When all goods produced in the economy are subject to a cash in advance constraint, the optimal amount of capital (as a factor of production) falls as inflation rises (Stockman, 1981; see also Orphanides and Solow, 1990). Inflation acts as an implicit tax over both consumption and investment.[1]

Other hypotheses also link higher rates of inflation to lower long term rates of growth. Shopping costs act in a very similar way to the cash in advance constraint (De Gregorio, 1992, 1993). Low inflation has also positive effects on the total level of employment through its effect on labour supply. That happens when leisure is in the utility function and there is a cash in advance constraint (Roldós, 1993), or again in the presence of shopping costs (De Gregorio, 1993).

A possible criticism to these arguments emerges because they resort in two potential unrealistic assumptions. The first is that at some stage money is necessary to buy all goods. As time passes, financial institutions evolve rapidly. Usually people acquire capital goods or durable goods through credit markets; the second argument is that people can choose the number of hours to work freely. Though that may be a tendency in the very long run, still now there are many institutional arrangements that preclude that possibility as significant.

Empirical observations might also be advocated to question the negative theoretical relation between the level of inflation and growth. For quite a long time, growth and inflation were both high in Brazil. In other countries (Mexico, Argentina) growth has increased during stabilisation programmes, but once inflation stabilises in lower levels, rates of growth fall. There seems to be more a relation between the reduction of inflation and growth than between the level of inflation and growth.

This paper asks whether in absence of the cash in advance constraint for capital goods, and under a utility function that does not depend upon leisure, some measures of inflation may still affect the long run rate of growth, or at least the long term output level.[2]

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[1] Hung (1993) extends the previous analysis in an endogenous growth model, concluding that higher levels of inflation are related to lower long term rates of growth of the per capita output.

[2] Quite a lot of the ideas of this paper were first treated in my PhD thesis for open economies with crawling peg exchange rate regimes.

The main results of the paper can be summarised as follows:

When there are incomplete credit markets because there is a cash in advance constraint on consumption goods exclusively, the optimal disinflation strategy is to reduce the rate of growth of prices gradually. There is a negative relation between the acceleration of inflation and growth.

The reason why growth and the acceleration of inflation are negatively related is that money and consumption are complements due to the cash in advance constraint. When inflation falls sluggishly, people increase their holdings of money in time since the shadow price of money (inflation) is also falling in time. Because consumption and money follow the same pattern, consumption is rising in time. Therefore, people reduce consumption in the present in order to increase gradually consumption in the future. That situation generates higher savings, more capital accumulation and higher growth.[3]

The discussed result contradicts the idea of maintaining a constant rate of creation of money (Friedman, 1968). Instead, the rate of growth of the supply of money has to fall also gradually. A gradual reduction of the rate of inflation is neither neutral nor superneutral and has permanent effects over output though it might be temporal.

The paper is divided in four sections: Section I sets an endogenous growth model; section II looks at the results of the government implementing different disinflation policies in the model; section III looks for the optimal disinflation strategy. Finally, section IV analyses the financial implications of some disinflation policies.

I. A Continuous Time Endogenous Growth Model

The presented model assumes a cash in advance constraint for consumption goods exclusively (Orphanides and Solow, 1990; Stockman, 1981)[4] and a modified version of the *AK* technology with an externality à la Romer (Rebelo, 1991a; Romer, 1986; De Gregorio, 1992).

[3] The mechanics of sluggish disinflation over consumption is very similar to Obstfeld's argument. The difference is that Obstfeld model is not a growth model (see Obstfeld, 1985).

[4] Orphanides and Solow (1990) and Stockman (1981) analyse the case of the cash in advance constraint affecting only consumption goods. In both studies, as this paper confirms, the result is that discrete changes in the rate of inflation do not affect the rate of growth of GDP. Lucas and Stockey (1987) also present a model where the purchases of some goods require the cash in advance constraint, while other goods may be purchased with credit cards.

There are two main agents in the economy: consumers (the private sector) and the government; and two factors of production: labour and capital. Given the technological conditions explained in the following pages, total production is exhausted in factor payments. The economy is closed and the government is the only agent that can print money, which is necessary to buy consumption goods.

I.1. Production

The production function of this economy is:

$$Y = AK^{\alpha} L^{1-\alpha} \tag{1}$$

Where Y is total production, K is the level of the capital stock and L is labour.

There is an externality à la Romer (Romer, 1986; De Gregorio, 1992), since:

$$A = A_1 K^{1-\alpha} \tag{2}$$

Where A_1 is a positive parameter.

Any firm working by itself observes (1), where A is a parameter for that particular firm. Nonetheless, when all firms increase the capital stock, the parameter A changes positively. In this way, the true production function for all the economy is:

$$Y = A_1 K L^{1-\alpha} \tag{3}$$

Production is linear with respect to capital. If labour is a parameter, the relevant framework is the AK model (see Rebelo, 1991a, 1991b).

Since every firm observes (1) instead of (2), the price of capital must be equal to a pseudo marginal productivity of capital:

$$\alpha AK^{\alpha-1} L^{1-\alpha} = \alpha A_1 L^{1-\alpha} = r \tag{4}$$

Where the term in the left-hand side is the pseudo marginal productivity of capital and r is the price of capital. It is noticeable that this measure does not depend on the quantity of capital. Therefore, higher

amounts of this factor on the economy do not reduce its perceived marginal productivity.[5]

Any particular firm also equates the marginal productivity of labour with its price (the wage). Then:

$$(1 - a)\ AK^{\alpha}L^{-\alpha} = (1 - \alpha)A_1 KL^{-\alpha} = W \qquad (5)$$

Where W is the real wage.

Increases in the capital stock have a positive linear relation with real wages.

By (4) and (5) it is possible to check that the sum $WL + rK$ is equal to the production function. The product is exhausted in factor payments. However, capital does not receive its own productivity but a smaller value due to the Romer externality.

I.2. Consumers

A representative individual maximises the intertemporal utility function.

$$Max \int_0^{\infty} U\left(C_t\right) \exp\left(-\theta t\right) dt \qquad (6)$$

$$C_t > 0 \qquad (69)$$

Where $U(.)$ is the instantaneous utility function, C is consumption, u **the subjective discount factor of the utility and t is time.**

People derive utility from consumption exclusively. For that reason they supply the maximum possible amount of labour L_0.

Maximisation is subject to the budget constraint.[6]

$$WL_0 + rK + rB + T - C - \pi m = Dm + DB + DK \qquad (7)$$

$$m_t > 0;\ K_t > 0 \qquad (79)$$

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[5] More capital does not affect the true marginal productivity of capital, either. This figure is given by the derivative of (3) with respect to capital and is

$$A_1 L^{1-\alpha}$$

[6] For convenience we have eliminated the subscript t everywhere, except when it is absolutely necessary.

Where
B: government bonds in real terms
T: net transfers of the government to the private sector
π: rate of inflation
m: real quantity of money
Dx: *dx/dt* for any variable *X*

Net incomes of people are used for consumption and savings. Savings take the form of shares of new capital, government bonds and money. Government bonds are supposed to be perfect substitutes of capital shares. That is why its real rate of return is equal to the pseudo marginal productivity of capital. Real money receives $-\pi$ as return. The reason why people keep money as an asset is a cash in advance constraint for consumption goods:[7]

$$m_t = \varphi C_t \tag{8}$$

Money is needed to buy consumption goods but not capital goods.[8]

A representative individual maximise (6) subject to (7), (8) and the restriction

$$V = m + B + K \tag{9}$$

Where *V* is the total private wealth.

First order conditions for this problem are:

$$U_c = \lambda(1 + i\varphi) = 0 \tag{10}$$

Where $i = r + \pi$

$$\frac{d\lambda}{dt} - \theta\lambda = -r\lambda \tag{11}$$

$$\lim_{t \to \infty} \lambda_t V_t \exp(-\theta t) = 0 \tag{12}$$

[7] The term φ is better understood in a discrete time model. The quantity of money hold by a person today may be spent in one year. Then $m_t = C_t$ if *t* represents a year. However, if *t* represents a month then $m_t = 12C_t$ because consumption in a month is twelve times smaller than consumption in a year. The fact that φ is a constant implies that qualitative results are the same whether it takes one value or another. Calvo (1986) uses the same specification for the cash in advance constraint.

[8] As long as the real rate of interest is greater than zero, (8) holds as an equality (see Calvo, 1986).

Combining (10) and (11) and rearranging[9]

$$DC = \frac{(\theta - r)U_c}{U_{cc}} + \frac{(Dr + D\pi)\varphi}{\left(1 + (r + \pi)\varphi\right)}\frac{U_c}{U_{cc}} \tag{13}$$

Where *DX* is *dX/dt* for any variable *X*. U_c is the marginal utility of consumption and U_{cc} is the first derivative of U_c with respect to consumption.

The ratio U_c/U_{cc} is known as the factor of risk aversion (Blanchard and Fischer, 1989). U_{cc} is negative since the marginal utility of consumption is decreasing. Thus, the acceleration of inflation has a negative effect over the rate of change of consumption.

Assuming an iso-elastic utility function:

$$U = \frac{C^{1-\frac{1}{\rho}}}{1 - \frac{1}{\rho}} \tag{14}$$

(Where ρ is the intertemporal elasticity of substitution of consumption.)

The factor of risk aversion is equal to –ρ*C*. For the same reason (13) becomes:

$$\frac{DC}{C} = \rho\left((r - \theta) - \frac{(Dr + D\pi)\varphi}{\left(1 + (r + \pi)\varphi\right)}\right) \tag{15}$$

The rate of growth of consumption depends negatively on the acceleration of the nominal interest rate *i*.

I.3. The Government

The government is supposed to spend in interest payments of the debt and net transfers to the private sector. It derives net incomes from the inflation tax. The government budget constraint is then:

$$T + rB - \pi m = Dm + DB \tag{16}$$

[9] Appendix 1 analyses the transversality condition (12) and the no-ponzi game condition suggested by Barro and Sala i Martin (1995, chapter 4).

Any excess of net expenditures (transfers plus interest payments on the domestic debt) over net incomes (the inflation tax) is covered issuing new domestic debt or real money.

I.4. Equilibrium

Since labour supply is inelastic, the above assumptions imply:

$$r = \alpha A_1 L_0^{1-\alpha} \tag{17}$$

Which means that the price of capital is fixed ($Dr = 0$). Therefore, if the utility function is iso-elastic, the rate of growth of consumption is:

$$\frac{DC}{C} = \rho\left((r - \theta) - \frac{\varphi D\pi}{\left(1 + (r + \pi)\varphi\right)}\right) \tag{18}$$

Combining the private budget constraint (7) with the government budget constraint, knowing that the product is exhausted in factor payments and rearranging:

$$Y - C = DK \tag{19}$$

Which is the national accounts identity. Output is distributed between consumption and investment in a closed economy.

Taking again the definition of the production, function (19) is transformed in:

$$A_1 L_0^{1-\alpha} K - C = DK \tag{20}$$

(20) is a differential equation in K. Together with (18) it constitutes a system for consumption and the stock of capital. Since at the end the price of capital does not depend upon its stock, the system is recursive. The acceleration of inflation enters in an exogenous way in the determination of C and K through its influence on the rate of growth of consumption.

II. Disinflation: Effects on Growth and Output

The way in which disinflation affects the behaviour of the capital stock and consumption in the pair of equations (18) and (20) depends upon the way in which it takes place.

A sudden unexpected change in the rate of inflation does not have any effect on the system, provided $D\pi$ is zero. In this case, since r and θ are constant, the rate of growth of consumption is also constant. The level of inflation does not matter and the economy will grow independently of it (see Stockman, 1981; Orphanides and Solow, 1990; and Hung, 1993). Nonetheless, there are other stabilisation policies that may generate higher growth.

II.1. Sluggish Disinflation and Balanced Growth

According to Obstfeld (1985), in Sidrausky type models (Sidrausky, 1967) sudden changes in the rate of inflation are neutral. However, sluggish disinflation may be non-neutral. Roldós (1993) shows that the Obstfeld argument may be extended to cash in advance models. This work tries to further extend these results to growth in cash in advance models.

A different way in which the government may generate higher growth through stabilisation is the following rule:

Between the interval of time $(0, \infty)$

$$\frac{\varphi D\pi}{\left(1+(r+\pi)\varphi\right)} = -\gamma;\ \gamma > 0 \tag{21}$$

Where γ is a positive constant.[10]

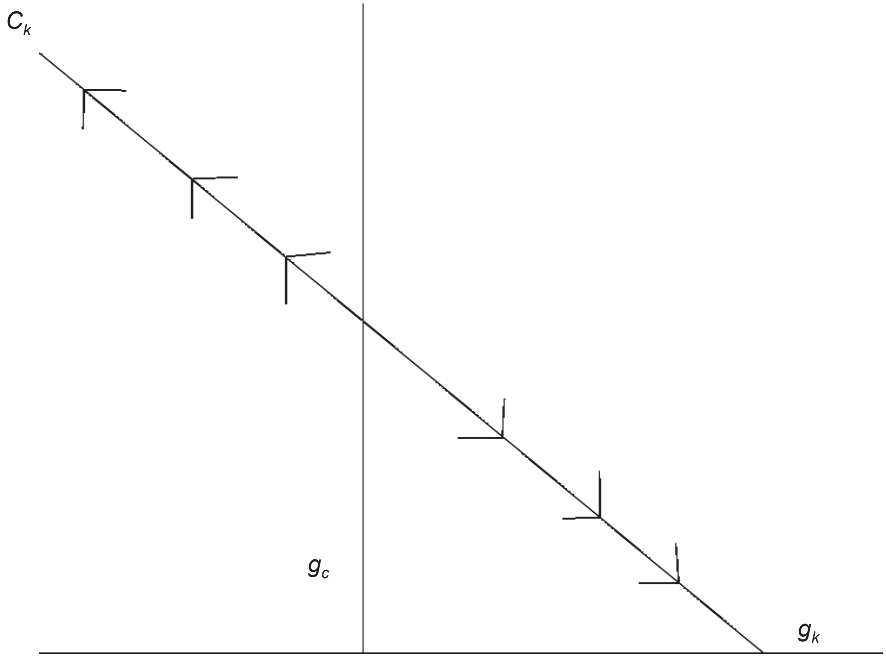
In this case, inflation follows the trajectory

$$\pi(t) = \left(\pi(0) + \left(\frac{1}{\varphi} + r\right)\right)\exp(-\gamma t) - \left(\frac{1}{\varphi} + r\right) \tag{22}$$

$$\lim_{t\to\infty} \pi(t) = -\left(\frac{1}{\varphi} + r\right) \tag{23}$$

[10] It is important to analyse the rule since next section will show that under some circumstances it may be optimal.

Figure 1



When time tends to infinity there is deflation in the economy. Under these assumptions, the behaviour of consumption is

$$\frac{DC}{C} = \rho\left(r - \theta + \gamma\right) \tag{24}$$

Consumption grows at a constant rate. The faster inflation falls, the greater is γ and the more consumption grows.

The simplest way to show that this rule produces higher growth than a constant rate of inflation is shown in Figure 1.

Figure 1 shows a line with negative slope, which can be derived dividing equation (20) by K:

$$A_1 L_0^{1-\alpha} - C_k = g_k \tag{25}$$

Where C_k is consumption divided by the stock of capital and g_k is the rate of growth of the capital stock. Since K and output behave in exactly the same way, g_k is also the rate of growth of output (g_y). For the same reason, C_k behaves as the ratio of consumption to GDP (C_y).

On the other hand, the vertical line represents equation (24), or the rate of growth of consumption (g_c), which is independent of the size of consumption and the capital stock.

Dynamics of this system is as follows:

If $g_c > g_k$, C_k is increasing and the economy is moving to the Northwest through the line (25). If $g_c < g_k$, C_k is falling and the economy is moving to the Southeast again through the line (25).

When both rates of growth are the same, the system is in equilibrium.

The system in figure 1 is very similar to the one studied by Barro and Sala i Martin (1995, p. 141-143). Their system might actually be represented by a diagram with the same dynamic characteristics shows in figure 1.

Though apparently unstable, the system in figure 1 has a saddle path property. A very important point is that equation (25) is a constraint in which the economy has to be always. For that reason, there are not predetermined variables in this model. If growth jumps, then C_k will also jump (see Sargent and Wallace, 1973; Blanchard and Fischer, 1989, pp. 239-245; and Drazen, 1985, for examples where dynamic systems are apparently globally unstable but where the fact that one or two variables are not predetermined precludes any systematic instability).

The reason why C_k may jump under unexpected changes in parameters is because though the capital stock K is a predetermined variable, consumption is not. If there is a sudden unexpected change in one of the parameters, consumption will change. The ratio C_k will also change in order to find a feasible solution. Figure 1 shows that solutions where the ratio C_k is always growing of falling may be ruled out. They quite possibly would break the transversality condition (12) but for sure they would break conditions (6') and (7'). Capital and/or consumption can not take negative values.[11]

Suppose that being in the equilibrium point with $\gamma = 0$ there is a sudden increase of this variable to a positive value. The line g_c shifts to the right at once. Taking the new equilibrium the point where the economy was originally is one where $g_c > g_k$ and then C_k is growing. But that can be neither an equilibrium nor a trajectory to a new equi-

[11] In the Ramsey model of exogenous growth, consumption is also a non-predetermined variable. It jumps to a new saddle path (the feasible solution) whenever there is any unexpected change in some parameter (see Barro and Sala i Martin, 1995, chapter 2; and Blanchard and Fischer, 1989, chapter 2).

librium, since C_k would be growing without bound. A continuous rise in consumption above the capacity of the economy would reduce the stock of capital to zero and to negative values, which is impossible. The only feasible solution is one where the growth of capital rises at the same time that the growth of consumption rises.

In this example, C_k falls at once, meaning that the level of consumption falls immediately, savings rise and then all real variables start growing at a higher rate. As in the Barro-Sala i Martin case, there is not transition dynamics under sudden unexpected changes in parameters (see Barro and Sala i Martin, 1995, pp. 142-143).

Since any permanent rise in γ produces this effect, this way of reducing inflation generates higher growth. The greater is γ, or better said the fastest inflation falls, the higher is growth.

The cash in advance constraint shows that consumption and money behave in the same way. Therefore, money grows at the rate of growth of GDP. A way in which the total government bonds also grow at the same rate is when the fiscal policy is such that:

$$\frac{T}{Y} = \frac{\pi m}{Y} = H \tag{26}$$

Where H is a constant value. Given the definition of the fiscal deficit in (16):

$$H = \frac{Dm}{m}\frac{m}{Y} + \frac{B}{Y}\frac{DB}{B} - r\frac{B}{Y} \tag{27}$$

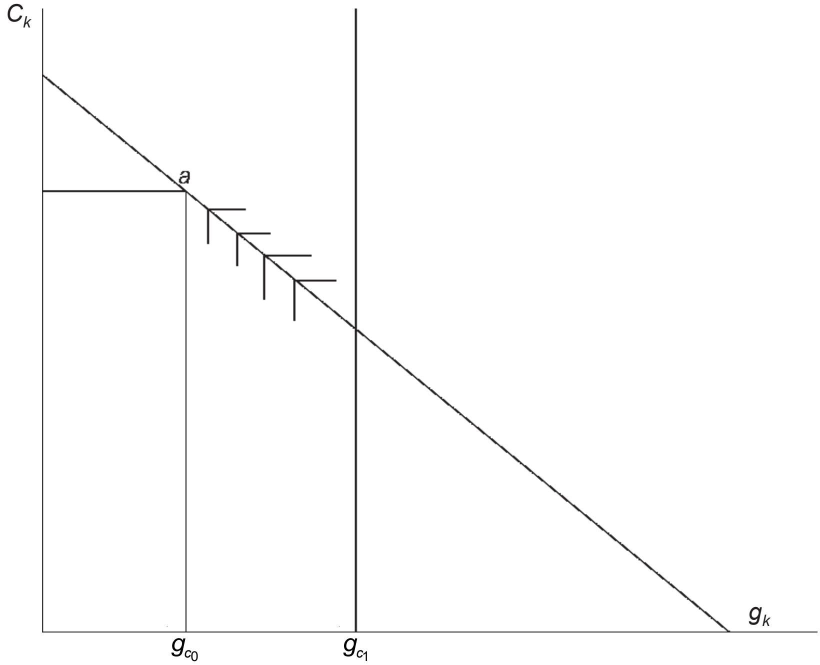
The first term of the right-hand side is constant. If H is also constant, the sum of the remaining terms in the right-hand side must be also constant. That only happens when bonds grow at the same rate than output. Therefore, if the government follows the policy suggested in (26) there is a complete balanced growth path.[12]

II.2. Sluggish Disinflation and Unbalanced Growth

The assumption that inflation falls forever could be quite unrealistic in the real world. Countries embodied in stabilisation programmes

[12] The rule is that the difference between net transfers and inflation tax must be proportional to output. Appendix 1 shows that still given this situation the rate of growth of the economy must be smaller than the rate of interest for the transversality condition (12) to hold.

Figure 2



usually pass through a period in which inflation is falling from one stable plateau to another, where its level is lower.

We will analyse one rule that makes easier the analysis of unbalanced growth.

The rule is the following:

During the interval (0, t_x) inflation is falling according to the rule

$$\frac{\varphi D\pi}{\left(1+\left(r+\pi\right)\varphi\right)}=-\gamma \tag{28}$$

During the interval (t_x, ∞) the reduction of inflation is zero.

For those reasons, in (0, t_x) the rate of growth of consumption is as (24), while during (t_x, ∞) it will be

$$\frac{DC}{C}=\rho\left(r-\theta\right) \tag{29}$$

Figure 2 shows this exercise.

Previously to time 0, inflation was zero and the economy was growing at the rate g_{c_0}. In the period (0, t_x), consumption is growing at the rate g_{c_1}. However, people expect a reversion of this policy. The final equilibrium must be at the original point *a*. The dynamic process already explained shows that instead of going to the new equilibrium point *b*, the economy goes to an intermediate point between g_{c_0} and g_{c_1}. Then it starts moving Northwest. The rate of growth of GDP rises first and then falls until it reaches the original level in t_x. At that precise moment inflation stops falling and reaches a new plateau. The line g_{c_1} shifts again and in a sudden way to g_{c_0} and the complete system is again in equilibrium. How much g_y rises at the beginning depends on the size of t_x. The greater is that time, the greater is the initial increase in the rate of growth.[13]

While the effects of this policy over the rate of growth of GDP and the relative size of consumption are temporal, the effects over output are permanent. The reason is simple: growth is either the original or greater. At the end, total consumption and output are greater than in absence of the policy.

The example of this sub-section shows the usefulness of having a model with some kind of "instability". In a globally stable model, expectations do not exert any influence in the endogenous variables of the model. Instead, in a model with saddle path properties, expectations always influence endogenous variables (see Sargent and Wallace, 1973, for examples of future expected policies).

Appendix 2 shows that other rules to reduce inflation sluggishly will also produce permanent effects on output. Next section will show that, in absence of subsidies to investment or capital, the rule to reduce inflation that produces a balanced growth path (equation 21) is optimal.

III. Disinflation, Growth and the Optimal Money Supply Rule

Is there an optimal disinflation strategy in this model?

To show that there is one, it is necessary to show first that growth is sub-optimal when inflation is constant.

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[13] A very important point is that consumption may jump in time zero but once it jumps at the very beginning it is predetermined. That is why the initial reduction in the ratio C_k takes place in a point between the original rate of growth g_{c0} and the transitory rate g_{c1}. If it were ot like that, the final solution would be unfeasible breaking either the transversality condition (12) or the fact that *K* and *C* have to be positive.

The way to do that is setting a hypothetical intertemporal planner. Since there is one representative individual in the economy, the objective of the planner is to maximise the intertemporal utility function (6) but now subject to the total constraint of the economy (20):

$$Max \int_0^{\infty} U(C_t) \exp(-\theta t)\, dt \tag{30}$$

Subject to

$$A_1 L_0^{1-\alpha} K - C = DK \tag{31}$$

The solution of this problem considering the iso-elastic utility function yields to the result

$$\frac{DC}{C} = \rho\left(A_1 L_0^{1-\alpha} - \theta\right) \tag{32}$$

Consumption grows at a constant rate. For the explanation already provided in the previous section, the optimal rate of growth is the one indicated by (32) in a balanced path.

The rate of growth of GDP with constant inflation is given by (29). Since $r = \alpha A_1 L_0^{1-\alpha}$, growth is suboptimal when inflation is constant.

The difference between optimal growth and growth under constant inflation is:

$$Dif = \rho(1-\alpha) A_1 L_0^{1-\alpha} \tag{33}$$

Therefore, the optimal rate at which inflation should be falling is:

$$\gamma = (1-\alpha) A_1 L_0^{1-\alpha} \tag{34}$$

What is the monetary rule consisting with this inflation reduction?

For the cash in advance constraint and the balanced growth path we know that

$$\frac{Dm}{m} = \frac{DC}{C} = g_y = g_k \tag{35}$$

Where g_y is the rate of growth of output.

Therefore, the optimal and consistent behaviour of the rate of growth of money in nominal terms is, taking the trajectory of inflation described in equation 22:

$$\mu_t = \left[\pi_0 + \left(\frac{1}{\varphi} + r\right)\right]\exp\left(-\gamma t\right) - \left(\frac{1}{\varphi} + r\right) + \rho\left(A_1 L_0^{1-\alpha} - \theta\right) \tag{36}$$

Where m is the rate of growth of nominal money.

$$\gamma = (1 - \alpha)A_1 L_0^{1-\alpha} \tag{37}$$

$$r = \alpha A_1 L_0^{1-\alpha} \tag{38}$$

When t tends to infinity

$$\mu(t) = (\rho - \alpha)A_1 L_0^{1-\alpha} - \left(\frac{1}{\varphi} + \theta\right) \tag{39}$$

At the very end, money in nominal terms may be growing or falling depending on the magnitudes. If ρ is smaller than α, μ will be negative and nominal money will be falling. Still for some values where ρ is greater than α, the rate of growth of nominal money is negative in the long run. It is only when ρ is sufficiently large when μ is always positive but falling.

IV. Financial Effects of the Optimal Money Supply Rule

The sluggish reduction of inflation produces interesting effects on the financial side:

Figure 1 shows a reduction in the value C_y when inflation starts falling sluggishly. Since capital and output are both predetermined values, the sudden reduction in C_y is accompanied by a corresponding reduction in consumption. Given the cash in advance constraint, real money also falls at once. Greater savings are accompanied by an initial portfolio movement from money to bonds. When the government accommodates, it will issue bonds at the beginning retiring money at the same time that it is reducing the rate of growth of money. If the government does not follow this policy, real money has to fall anyway and the price level increases at once.

This last result contrasts with some other in the literature (see for example Sargent and Wallace, 1973; and Buiter and Miller, 1985), in which the reduction of inflation produces an initial reduction on the price level.

Concluding Remarks

This paper shows that when credit markets are incomplete and money is necessary to consume, but not to invest, sluggish disinflation has permanent and positive effects on output, growth or both. The article shows that in the presence of externalities there is an optimal disinflation policy. In this case, the consistent monetary policy is to reduce the growth of nominal money supply sluggishly.

When inflation falls at once and in a permanent way the shadow price of consumption remains constant from that moment onwards. Therefore, there are not incentives to shift consumption in time and nothing happen to savings and growth. Instead, when inflation falls sluggishly, the perceived shadow price of future consumption is lower. There is an incentive to substitute present by future consumption. As long as inflation is always falling, future consumption is always growing. That means more savings today and higher growth.

The policy of reducing inflation gradually may be optimal when there is an externality à la Romer. In that case, growth is insufficient from a social point of view. The perceived marginal productivity of capital is smaller than the true productivity. A gradual reduction of inflation helps to increase savings and to overcome the externality.

In absence of externalities (e.g. Rebelo, 1991a, 1991b), growth would be optimal under constant inflation independently of its size. In that case inflation should be constant to maximise social welfare.

Under the Romer externality, alternative optimal policies to improve welfare would be subsidies to investment or reducing indirect taxes to consumption gradually (e.g. VAT). This last policy would have a similar effect than reducing inflation sluggishly, since the shadow price of consumption would be falling in time, generating higher savings in the present.

Appendix 1. On the Feasibility of Balanced Growth Paths

Proposition: In every circumstance a balanced growth path is feasible when the real rate of interest is greater than the rate of growth of GDP.

Proof:

The transversality condition is, repeating (12) for convenience:

$$\lim_{t \to \infty} \lambda_t V_t \exp(-\theta t) = 0 \qquad \text{(A.1.1)}$$

Because of first order conditions and given the iso-elastic utility function:

$$\lambda_t V_t \exp(-\theta t) = \frac{C_t^{-\frac{1}{\rho}} V_t \exp(-\theta t)}{(1 + (r + \pi)\varphi)} \qquad \text{(A.1.2)}$$

$C^{1/\rho}$ is the marginal utility of consumption U_c.
Using (22) and the balanced growth assumption, we get:

$$\lambda_t V_t \exp(-\theta t) = \frac{V_c \exp(-\theta t)}{C^{\frac{1}{\rho}-1}(\varphi\pi(0) + r\varphi + 1) \exp(-\gamma t)} \qquad \text{(A.1.3)}$$

V_c is V/C a constant term because the balanced growth assumption.[14]

But

$$C_t = C_i \exp(g_y t) \qquad \text{(A.1.4)}$$

Where g_y is the rate of growth of GDP, DY/Y.
Hence:

$$\lambda_t V_t \exp(-\theta t) = \frac{V_c \exp\left(\left(\gamma - \theta - \left(\frac{1}{\rho} - 1\right) g_y\right) t\right)}{C_i^{\frac{1}{\rho}-1}(\theta\pi(0) + 1 + r\varphi)} \qquad \text{(A.1.5)}$$

For the transversality condition to hold, it is necessary:

$$\gamma - \theta - \left(\frac{1}{\rho} - 1\right) g_y < 0 \qquad \text{(A.1.6)}$$

Substituting $g_y = \rho(r + \gamma - \theta)$ and rearranging:

$$\alpha A_1 > \rho(\alpha A_1 + \gamma - \theta) \qquad \text{(A.1.7)}$$

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[14] It is assumed here that the government follows the fiscal policy suggested in (26).

or

$$r > g_y \tag{A.1.8}$$

This appendix also analyses a no-ponzi game condition suggested by Barro and Sala i Martin (1995, chapter 4, p. 141). The result is the same: balanced growth is feasible whenever the real rate of interest is greater than the rate of growth of the economy.

The no-ponzi game condition is:

$$\lim_{t\to\infty} V_t \exp\left(-\int_0^t r(h)dh\right) = 0 \tag{A.1.9}$$

Since in the model studied in this paper r is a constant term, the condition can be transformed in:

$$\lim_{t \to \infty} V_t \exp(-rt) = 0 \tag{A.1.10}$$

In a context of complete balanced growth

$$V_t = V(0) \exp(\rho(r - \theta + g)t) \tag{A.1.11}$$

Substituting this value in the no-ponzi game condition (A.1.10) and rearranging, that condition becomes:

$$\lim_{t \to \infty} V(0) \exp((\rho(r - \theta + \gamma) - r)\ t) \tag{A.1.12}$$

The condition holds if

$$r > \rho(r - \theta + \gamma) = g_y \tag{A.1.13}$$

Appendix 2. Growth and other Disinflation Strategies

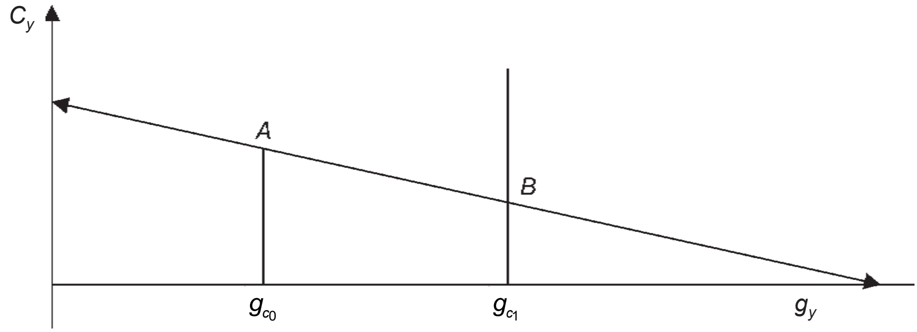
Suppose that inflation follows the trajectory

$$\pi = \pi_0 e^{-bt} \tag{A.2.1}$$

Where b is a positive constant.

This trajectory implies inflation converging exponentially to zero as time passes and could be considered as more realistic than other disinflation strategies.

Figure A.2



Under this policy and assuming again the iso-elastic utility function, the growth of consumption is defined by:

$$\frac{DC}{C} = \rho\left(r - \theta + \frac{b\pi_0 e^{-bt}\varphi}{\left(1 + \left(r + \pi_0 e^{-bt}\right)\varphi\right)}\right) \tag{A.2.2}$$

The term

$$\frac{b\pi_0 e^{-bt}\varphi}{\left(1 + \left(r + \pi_0 e^{-bt}\right)\varphi\right)} = \tau \tag{A.2.3}$$

Falls at time passes, since:

$$\frac{\partial\tau}{\partial t} = \frac{-\varphi b^2 \pi_0 e^{-bt}\left(1 + r\varphi\right)}{\left(1 + \left(r + \pi_0 e^{-bt}\right)\varphi\right)^2} < 0 \tag{A.2.4}$$

Figure A.2 shows the effects of this policy.

Originally the economy is in point A, where $g_{y_0} = g_{c_0}$. In time zero, inflation starts falling in the way described by (A.2.1). The line g_c shifts to the right, to g_{c_1} and then it gradually moves to the left, reaching again g_{c_0} when t tends to infinity. The mathematical solution for g_y is, perhaps, very complicated but an heuristic argument is good to guess its trajectory:

Once the line g_c moves to g_{c_1}, g_y can be neither in A nor to the Northwest of A. If that were the case, and given the dynamics shown by the arrows, the economy could never return to equilibrium and the solution would be unfeasible, with growth falling forever and consumption growing without bound. For the same reason g_y can not be at the Southeast of point *B*. Therefore at the moment in which g_c moves, g_y has to move to a point between g_{c_0} and g_{c_1} and move gradually to the Northwest. As *t* tends to infinity, g_y tends to g_{c_0} again.

The trajectory of growth has to be similar, if not equal, to the trajectory of the growth of consumption. It will rise in a sudden way at time zero and then it gradually will fall to its original level in the very long run. Disinflation in this context has a positive temporal effect over growth and a long run permanent effect on output.

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