On Consumption, Investment and Risk

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Abstract: The Mexican episode of 1992-1994 was characterized by a steep rise in consumption accompanied by a sharp fall in investment. This paper provides an explanation of the negative response of investment to political risk, as occurred in Mexico between 1992 and 1994. It is assumed that, inside an adjustable band, the expected rate of depreciation is driven by a mixed diffusion-jump process and the expected real rate of return on an international bond is governed by a diffusion process, both processes being correlated. This paper analyzes a small open stochastic economy. Two cases are considered: *i*) a cash-in-advance, Ramsey-type economy, and *ii*) a Sidrauski-type economy.

Resumen: La experiencia mexicana de 1992-1994 estuvo caracterizada por un marcado incremento en el consumo, acompañado de una fuerte caída en la inversión. Este trabajo proporciona una explicación de la respuesta negativa de la inversión al riesgo político, como ocurrió en México entre 1992 y 1994. El trabajo supone que, dentro de una banda ajustable, la tasa esperada de depreciación sigue un proceso mixto de difusión con saltos y el retorno real esperado de un bono internacional sigue un proceso de difusión; los dos procesos se encuentran correlacionados entre sí. Este trabajo analiza una economía estocástica, pequeña y abierta. Se consideran dos casos: *i*) una economía "*cash-in-advance*" del tipo de Ramsey y *ii*) una economía del tipo de Sidrauski.

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1. Introduction

Between 1992 and 1994 (up until the December 1994 financial debacle) Mexico had an intense trade opening, a high degree of capital mobility, and a strong privatization program. The above actions were based on an inflation stabilization plan in which the rate of depreciation was set as a nominal anchor. However, unexpected political risk arose from a weak "Pacto"; violence in Chiapas; the assassination of a presidential candidate; recommendations from specialists to devaluate; the beginning of a banking system crises; and proximate elections. As a consequence of the above circumstances, the intended exchange-rate policy was expected to have no continuity. Investors were uncertain of the commitment of the government to defend the announced exchange-rate policy, and future deviations, including devaluations, were expected. The anticipated temporariness of the plan to stabilize the exchange rate led to an increase in consumption, which in turn caused a fall in investment.

The purpose of this paper is to examine the impact of political risk on consumption and investment. We achieve this by modeling the stochastic return on an international bond by a diffusion (Markovian) process, and the random variations in the expected rate of depreciation by a mixed diffusion-jump process (*i.e.*, the Brownian component guides the depreciation rate, and the Poisson component governs possible devaluations). The central experiment of the paper consists of setting initially both the rate of depreciation and the probability of devaluation (*i.e.*, a jump in the exchange rate) at a low level. Subsequently, due to a climate of political uncertainty, the exchange-rate policy is expected to be abandoned, and so both the rate of depreciation and the probability of devaluation are expected to be higher.

There is an increasing amount of literature in international finance using expected utility maximization with a diffusion-jump process driving the dynamics of the exchange rate. For examples see Svensson (1992), and Penati and Pennacchi (1989). Other illustrations of applications of mixed diffusion-jump processes to finance can be found in Ahn and Thompson (1988), Jarrow and Rosenfeld (1984), Malliaris and Brock (1982), and Merton (1976 and 1971).

The organization of this paper is as follows. In the next section, we deal with a Ramsey-Cass-Koopmans stochastic economy subject to a cash-in-advance constraint. Here, we study the impact of political risk on investment. In section 3, we extend the model to a Sidrauski-type stochastic economy. Finally, in section 4, we give a summary of the main findings and make the conclusions. Three appendices contain some technical details about the investor's decision problem.

2. Exchange Rate and Rate of Return Dynamics

In order to generate solutions which are analytically tractable the structure of the economy will be kept as simple as possible. The main assumptions in our stylized model of a small open stochastic economy will be that the investor perceives that: *1)* the expected rate of depreciation follows a mixed diffusion-jump process, and *2)* the rate of return of an international bond is driven by a diffusion process, where both processes are correlated.

2.1. Assets

Let us consider a small open stochastic economy with a representative infinitely lived investor in a world with a single perishable consumption good. The good is freely traded at a domestic price P_t, determined by the purchasing power parity condition, $P_t = P_t^* E_t$, where P_t^* is the dollar price of the good in the rest of the world, and E_t is the nominal exchange rate. It will be assumed, from now on, that P_t^* is fixed and for simplicity equal to 1, which readily implies that the price level, P_t, is equal to the exchange rate, E_t. The initial value E_0 is supposed to be known.

The ongoing uncertainty about the dynamics of the expected rate of depreciation is driven by a mixed diffusion-jump process. In such a case, we suppose that the representative investor perceives that the expected inflation rate, $\mathrm{d}P_t/P_t$, and thus the expected rate of depreciation, $\mathrm{d}E_t/E_t$, is driven by a geometric Brownian motion with Poisson jumps:

$$\frac{\mathrm{d}P_t}{P_t} = \frac{\mathrm{d}E_t}{E_t} = \varepsilon \mathrm{d}t + \sigma \mathrm{d}z_t + \nu \mathrm{d}q_t \tag{1}$$

satisfying, inside an adjustable band,

$$E_0(1+\nu)^{q_t} B_t \leq E_t \leq E_0(1+\nu)^{q_t} A_t, \tag{2}$$

where the drift ε is the mean expected rate of depreciation conditional on no jumps, σ is the instantaneous volatility of the expected rate of depreciation, ν is the mean expected size of upward jumps in the exchange rate, and z_t is a standard Wiener process, that is, $\mathrm{d}z_t$ is a temporally independent normally distributed random variable with $\mathrm{E}[\mathrm{d}z_t] = 0$ and $\mathrm{Var}[\mathrm{d}z_t] = \mathrm{d}t$. The number of devaluations per unit of time occurs according to a Poisson process q_t with intensity λ, so Pr{one unit jump during $\mathrm{d}t$} = $\Pr\{\mathrm{d}q_t = 1\} = \lambda \mathrm{d}t + o(\mathrm{d}t)$, whereas Pr{no jump during $\mathrm{d}t$} = $\Pr\{\mathrm{d}q_t = 0\} = 1 - \lambda \mathrm{d}t + o(\mathrm{d}t)$. We initially set $q_0 = 0$. Moreover, processes $\mathrm{d}z_t$ and $\mathrm{d}q_t$ are assumed to be correlated. The adjustment stochastic processes A_t and B_t, linked with monetary policy, satisfy $B_t < A_t$ and $A_t > 0$ with probability one. The normalized exchange rate $e_t = E_t / E_0(1+\nu)^{q_t}$, inside the band $B_t \leq e_t \leq A_t$, will obey $\mathrm{d}e_t/e_t = (\mathrm{d}E_t/E_t) - \nu \mathrm{d}q_t$, from where

$$\frac{\mathrm{d}e_t}{e_t} = \varepsilon \mathrm{d}t + \sigma \mathrm{d}z_t. \tag{3}$$

Because of the specific interest of this paper in once-and-for-all changes in the rate of depreciation and in the intensity parameter, we assume that ε, σ, ν and λ are all positive constants.

For the sake of simplicity, we leave out from our analysis a riskless asset. Investors will hold two real assets: real cash balances, $m_t = M_t/P_t$, where M_t is the nominal stock of money, and an international bond, b_t. Thus, the investor's real wealth, W_t, is defined by

$$W_t = m_t + b_t, \tag{4}$$

where the initial wealth, W_0, is exogenously determined. Furthermore, we suppose that the rest of the world does not hold domestic currency (*i.e.*, the peso is not an asset for foreigners). The stochastic dynamics of the real rate of return on bonds evolves in accordance with

$$\mathrm{d}r_t = r_o r_t \mathrm{d}t + \sigma_o r_t \mathrm{d}x_t, \tag{5}$$

where the drift, r_o, is the mean rate of return, σ_o is the instantaneous volatility of the expected rate of return, and x_t is a standard Wiener process; *i.e.*, $\mathrm{d}x_t$ is a temporally independent normally distributed random variable with $\mathrm{E}[\mathrm{d}x_t] = 0$ and $\mathrm{Var}[\mathrm{d}x_t] = \mathrm{d}t$. Moreover, we suppose that $\mathrm{d}z_t$ and $\mathrm{d}x_t$ are correlated, so

$$\mathrm{d}z_t \mathrm{d}x_t = \frac{\mathrm{Cov}(\mathrm{d}x_t,\ \mathrm{d}z_t)}{\sigma_o \sigma} \mathrm{d}t, \tag{6}$$

where $\mathrm{Cov}(\mathrm{d}x_t, \mathrm{d}z_t)$ is the covariance between $\mathrm{d}x_t$ and $\mathrm{d}z_t$. We will assume that disturbances in the return rate and the exchange rate are positively correlated (as observed in Mexico during 1992-1994), that is, $\mathrm{Cov}(\mathrm{d}x_t, \mathrm{d}z_t) > 0$.

If capital is perfectly mobile, the real domestic interest rate, defined as $(\mathrm{d}R_t / R_t) - (\mathrm{d}E_t / E_t)$, must be equal to $\mathrm{d}r_t / r_t$ over any instant. Consequently, the expected nominal interest rate is given by

$$\frac{\mathrm{d}R_t}{R_t} = i\mathrm{d}t + \sigma \mathrm{d}z_t + \sigma_o \mathrm{d}x_t + \nu \mathrm{d}q_t,$$

where

$$i = r_o + \varepsilon \tag{7}$$

is the mean expected nominal interest rate conditional on no jumps.

Consider a Clower-type constraint of the form, $m_t \geq \alpha c_t$, where c_t is consumption and $\alpha > 0$ is the time that money must be held to finance consumption. Given that $i > 0$, the investor has incentive to hold only

$$m_t = \alpha c_t. \tag{8}$$

The stochastic rate of return of holding real cash balances, $\mathrm{d}R_M$, is simply the percentage change in the inverse of the price level. By applying the generalized Itô's lemma for diffusion-jump processes to the inverse of the price level with (1) as the underlying process (see Appendix III, formula (III.1)), we obtain

$$\mathrm{d}R_M = P_t \mathrm{d}\left(\frac{1}{P_t}\right) = \left(-\varepsilon + \sigma^2 \mathrm{d}t\right) - \sigma \mathrm{d}z_t + \left(\frac{1}{1+\nu} - 1\right) \mathrm{d}q_t. \tag{9}$$

2.2. Investor's Portfolio Problem

The stochastic investor's wealth accumulation in terms of the portfolio shares, $\omega_t = m_t / W_t$, $1 - \omega_t = b_t / W_t$, and consumption, c_t (the numeraire good), is determined by the following system of stochastic differential equations:

$$
\begin{cases}
\dfrac{\mathrm{d}W_t}{W_t} = \omega_t \dfrac{\mathrm{d}Q_t}{Q_t} + (1-\omega_t)\dfrac{\mathrm{d}r_t}{r_t} - \dfrac{c_t}{W_t}\mathrm{d}t, \\
\dfrac{\mathrm{d}Q_t}{Q_t} = (-\varepsilon + \sigma^2)\mathrm{d}t - \sigma \mathrm{d}z_t + \left(\dfrac{1}{1+\nu} - 1\right)\mathrm{d}q_t, \\
\dfrac{\mathrm{d}r_t}{r_t} = r_o \mathrm{d}t + \sigma_o \mathrm{d}x_t, \\
\dfrac{c_t}{W_t} = \alpha^{-1}\omega_t,
\end{cases}
\tag{10}
$$

where $Q_t = 1/P_t$ is the price of money in terms of goods. To avoid unnecessary technical complications, we exclude the investor real wage from the analysis. By solving system (10) in terms of $\mathrm{d}W_t/W_t$, we get

$$
\mathrm{d}W_t = W_t\left[(r_o - \rho\omega_t)\mathrm{d}t - \omega_t(\sigma \mathrm{d}z_t + \sigma_o \mathrm{d}x_t) + \sigma_o \mathrm{d}x_t + \left(\frac{1+\nu(1-\omega_t)}{1+\nu} - 1\right)\mathrm{d}q_t\right], \tag{11}
$$

where $\rho = \alpha^{-1} + i - \sigma^2$. Our analysis will be only concerned with small values of the total volatility compared with the mean expected rate of depreciation in such a way that

$$
\beta \equiv \varepsilon - \sigma^2 - \sigma_o^2 > 0. \tag{12}
$$

The competitive, risk-averse investor derives utility from consumption, c_t, and wishes to maximize her/his overall discounted, Von Neumann-Morgenstern utility, at time $t = 0$, given by

$$
V_0 = E_0 \int_0^\infty \log(c_t) e^{-r_o t} \mathrm{d}t, \tag{13}
$$

where E_0 is the conditional expectation on using all available information at $t = 0$. In order to generate closed-form solutions, we have chosen the logarithmic utility function.

In maximizing (13) subject to the wealth constraint as given in (11), the first-order condition for an interior solution is (see Appendix I)

$$
\frac{r_o}{\omega} - \frac{\lambda\nu}{1+\nu(1-\omega)} = A + \omega B, \tag{14}
$$

where $A = \rho - \sigma_o^2 - \mathrm{Cov}(\mathrm{d}x_t, \mathrm{d}z_t)$, and $B = \sigma^2 + \sigma_o^2 + 2\mathrm{Cov}(\mathrm{d}x_t, \mathrm{d}z_t) > 0$. We have not imposed any positivity constraint of the form $\omega_t \geq 0$, so unrestricted short sales are permitted. In what follows, without loss of generality, we will suppose that $\mathrm{Cov}(\mathrm{d}x_t, \mathrm{d}z_t)$ is bounded from above so that

$$0 < \mathrm{Cov}(\mathrm{d}x_t, \mathrm{d}z_t) < \beta. \tag{15}$$

From (15), we immediately find that $A > 0$. Observe that (14) is a cubic equation with one negative and two positive roots, and only one root satisfying $0 < \omega^* < 1$. To see this graphically, let us define the left-hand side of (14) by

$$f(\omega) = \frac{r_o}{\omega} - \frac{\lambda\nu}{1 + \nu(1 - \omega)}. \tag{16}$$

Function $f(\omega)$ has the following properties:

$$f(0^+) = +\infty,\ f(0^-) = -\infty,\ f(+\infty) = 0,\ f(-\infty) = 0,\ f(1) = r_o - \lambda\nu,$$
$$f\left(\frac{r_o}{r_o + \lambda}\left(1 + \frac{1}{\nu}\right)\right) = 0,\ f\left(\left(1 + \frac{1}{\nu}\right)^+\right) = +\infty,\ f\left(\left(1 + \frac{1}{\nu}\right)^-\right) = -\infty$$

With this information, we can sketch the graph of $f(\omega)$ in Figure 1. From the definition of A and β, and from (15), it is straightforward to see that $A > r_0 - \lambda\nu$, as shown in Figure 1. The straight line $A + \omega B$, defined by the right-hand side of (14), intersects the graph of $f(\omega)$ three times, one of which produces $\omega^* \in (0, 1)$. We rule out from our analysis the other two solutions to (14); a negative root would imply negative consumption, whereas a root greater than one would not be feasible since the investor has incentive to minimize her/his holdings of real balances.

Figure 1. Determination of Optimal ω^*

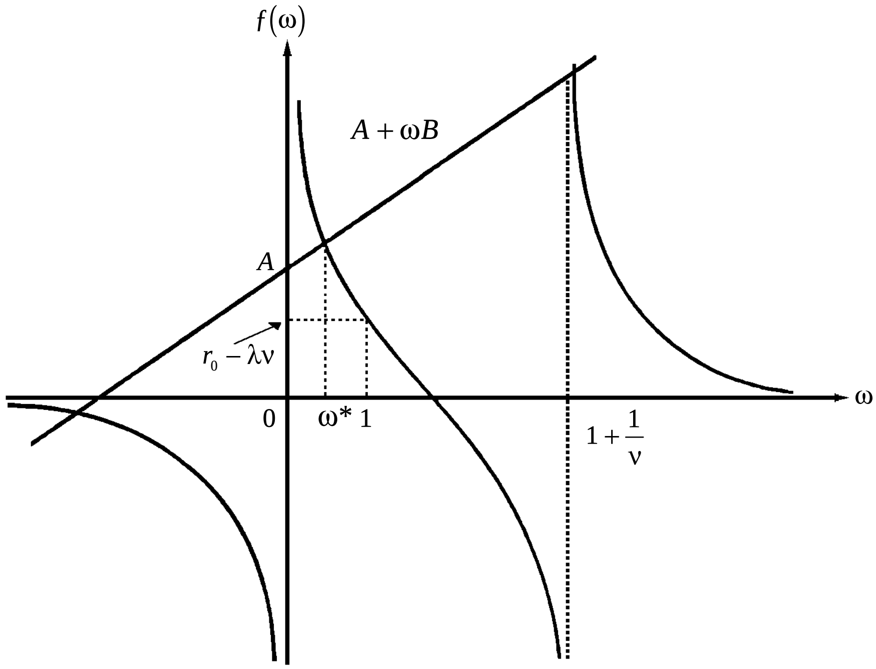


Figure 1 depicts the determination of the optimal ω^* for the case $r_o - \lambda\nu > 0$. The reader can see by inspecting Figure 1 that when $r_o - \lambda\nu < 0$, the optimal share ω^* will still be in (0, l).

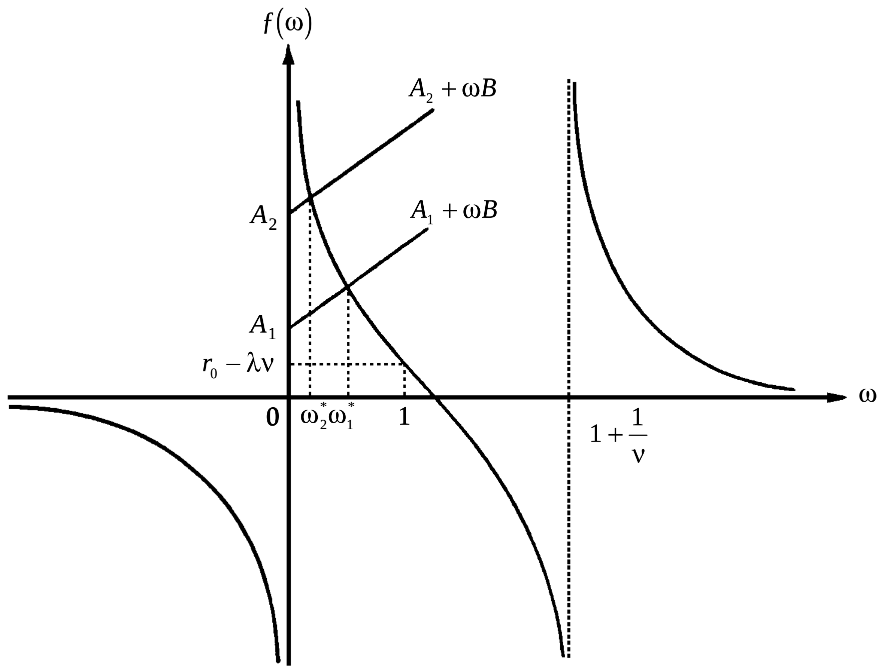
We harvest now a couple of important results:

Proposition 1. A once-and-for-all increase in the rate of depreciation, which results in an increase in the future opportunity cost of purchasing goods, leads to an increase in the proportion of wealth devoted to present consumption, which in turn decreases the corresponding proportion allocated for present investment.

It is enough to observe that $\varepsilon_1 < \varepsilon_2$ implies $A_1 < A_2$, which shifts the line $A + \omega B$ upward, reducing the equilibrium value of ω^*, as depicted in Figure 2, from where the claim stated in Proposition 1 readily follows. Alternatively, we may differentiate (14) to obtain

$$\frac{\partial \omega^*}{\partial \varepsilon} = -\left[\frac{r}{(\omega^*)^2} + \frac{\lambda\nu^2}{\left[1 + \nu(1 - \omega^*)\right]^2} + B\right]^{-1} < 0.$$

Figure 2. Response of ω* to Once-and-for-all Changes in ε



Proposition 2. A once-and-for-all increase in the expected number of devaluations per unit of time will increase the future opportunity cost of purchasing goods, which in turn increases the proportion of wealth set aside for present consumption. The effect on the proportion of wealth allocated for present investment is similar to that stated in Proposition 1.

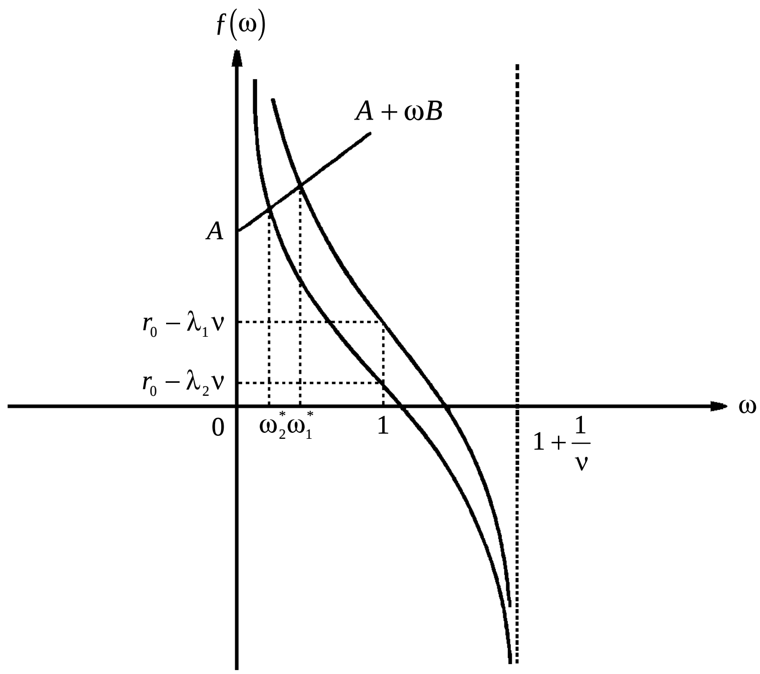
As depicted in Figure 3, an increase in λ, from λ_1 to λ_2, will shift $f(\omega)$ downward decreasing the value of ω^*, from where the proof of the above proposition follows. Alternatively, differentiating (14), we get

$$\frac{\partial \omega^*}{\partial \lambda} = \frac{\nu}{1+\nu(1-\omega^*)}\left(\frac{\partial \omega^*}{\partial \varepsilon}\right) < 0.$$

Thus, the elasticity of the rate of depreciation with respect to the probability of devaluation satisfies:

$$\frac{\dfrac{\partial \omega^*}{\partial \varepsilon}}{\dfrac{\partial \omega^*}{\partial \lambda}} > 1.$$

Figure 3. Response of ω* to Once-and-for-all Changes in λ



3. Welfare Implications

We assess now the magnitudes of the impacts on welfare of once-and-for-all changes in the mean expected rate of depreciation and in the probability of devaluation. As usual, the welfare criterion, *W*, of the representative investor is the maximized utility starting from the initial real wealth, W_0. Therefore, economic welfare is given by (see Appendix I, formula (I.3))

$$\mathrm{W}(\varepsilon,\lambda;W_0)\equiv I(W_0,0)=\frac{1}{r_o}\left[1+\log(W_0)+\log(\alpha^{-1}\omega^*)\right]$$
$$-\frac{1}{r_o^2}\left[A\omega^*+\frac{1}{2}\left[(\omega^*)^2B+\sigma_o^2\right]-\lambda\log\left(\frac{1+\nu(1-\omega^*)}{1+\nu}\right)\right]. \qquad (17)$$

A routine exercise of comparative statics leads to:

$$\frac{\partial W}{\partial \varepsilon} = -\frac{\omega^*}{r_o^2} < 0, \tag{18}$$

and

$$\frac{\partial W}{\partial \lambda} = \frac{1}{r_o^2}\log\left(\frac{1+\nu(1-\omega^*)}{1+\nu}\right) < 0, \tag{19}$$

As it might be expected, welfare behaves as a decreasing function of both the mean expected rate of depreciation and the probability of devaluation. The critical assumption of logarithmic utility accounts for such results.

4. Wealth, Consumption, and Dynamic Implications

The stochastic process that generates wealth when the optimal rule is applied will be now derived. After substituting the optimal share ω^* into (11), we get

$$\begin{aligned} \mathrm{d}W_t = W_t\Bigg[&\left(\frac{\lambda\nu\omega^*}{1+\nu(1-\omega^*)} + B(\omega^*)^2 - \left[\sigma_o^2 + \mathrm{Cov}(\mathrm{d}x_t, \mathrm{d}z_t)\right]\omega^*\right)\mathrm{d}t \\ &- \omega^*\sigma \mathrm{d}z_t + (1-\omega^*)\sigma_o \mathrm{d}x_t + \left(\frac{1+\nu(1-\omega^*)}{1+\nu} - 1\right)\mathrm{d}q_t\Bigg]. \end{aligned} \tag{20}$$

The solution to the above stochastic differential equation, conditional on W_0, is given by (see Appendix III, formula (III.2))

$$W_t = W_0 e^{\delta_t(\omega^*)}, \tag{21}$$

where

$$\delta_t(\omega^*) = \eta_t(\omega^*) + \gamma_t(\omega^*),\ \eta_t \sim \mathrm{N}\left(F(\omega^*)t, G(\omega^*)t\right),\ \gamma_t = H(\omega^*)q_t, \\ \text{and } q_t \sim \mathrm{P}(\lambda t), \tag{22}$$

and the stationary components of the parameters of the above distributions are:

$$F(\omega^*) = \frac{\lambda\nu\omega^*}{1+\nu(1-\omega^*)} + \frac{1}{2}\left[(\omega^*)^2 B - \sigma_o^2\right],$$

$$G(\omega^*) = (\omega^*)^2 B + \sigma_o^2 - 2\omega^*\left[\sigma_o^2 + \mathrm{Cov}(\mathrm{d}x_t, \mathrm{d}z_t)\right] > 0,$$

and

$$H(\omega^*) = \log\left(\frac{1+\nu(1-\omega^*)}{1+\nu}\right) < 0.$$

Notice that

$$\mathrm{E}\left[\delta_t(\omega^*)\right] = \left[F(\omega^*) + H(\omega^*)\lambda\right] t$$

$$\text{and } \mathrm{Var}\left[\delta_t(\omega^*)\right] = \left[G(\omega^*) + \left[H(\omega^*)\right]^2 \lambda\right] t. \tag{23}$$

Though $F(\omega^*)$ and $H(\omega^*)$ have opposite signs, $\mathrm{E}\left[\delta_t(\omega^*)\right]$ remains positive. Indeed, since $x - 1 - \log(x) \geq 0$ holds for all $x > 0$,

$$\frac{\nu\omega^*}{1+\nu(1-\omega^*)} - \log\left(\frac{1+\nu}{1+\nu(1-\omega^*)}\right) \geq 0,$$

from where the claim about the sign of $\mathrm{E}\left[\delta_t(\omega^*)\right]$ readily follows.

The stochastic process for consumption can be written, from (8) and (21), as

$$c_t^* = \alpha^{-1}\omega^* W_0 e^{\delta_t(\omega^*)}. \tag{24}$$

It is worthwhile to note that in the stochastic framework, we cannot determine the level of consumption. We can only compute the probability that, at a given time, a certain level of consumption occurs. Moreover, the assumption that the investor's time preference rate is equal to the real interest rate does not ensure a steady-state level of consumption.

5. Money in the Utility Index

The cash-in-advance assumption is somewhat restrictive in the sense that money is only seen as medium of exchange. We ease this assumption by including currency directly in the utility function because of its liquidity services. In such a case, the stochastic wealth accumulation in terms of the portfolio shares and consumption becomes

$$\mathrm{d}W_t = W_t\Big[\big(r_o - \phi\omega_t\big)\mathrm{d}t - \omega_t\big(\sigma \mathrm{d}z_t + \sigma_o \mathrm{d}x_t\big) + \sigma \mathrm{d}x_t$$

$$+\left(\frac{1+\nu(1-\omega_t)}{1+\nu} - 1\right)\mathrm{d}q_t\Big] - c_t \mathrm{d}t, \tag{25}$$

where $\phi = i - \sigma^2$.

The expected utility at time $t = 0$, V_0, now takes the form:

$$V_0 = \mathrm{E}_0 \int_0^\infty \big[\theta \log(c_t) + \log(m_t)\big] e^{-r_o t}\mathrm{d}t, \tag{26}$$

We have chosen again the logarithmic utility function to generate closed-form solutions.

The first order conditions for an interior solution to the problem of maximizing (26) subject to (25) are given by (see Appendix II)

$$c_t = \frac{\theta r_o}{1+\theta} W_t \quad \text{and} \quad \frac{r_t}{(1+\theta)\omega} - \frac{\lambda\nu}{1+\nu(1-\omega)} = D + \omega B, \tag{27}$$

where $D = \phi - \sigma_o^2 - \mathrm{Cov}(\mathrm{d}x_t, \mathrm{d}z_t)$ and B is taken as in section 2. The second equation above is similar to that of (14), except for the factor $1/(1+\theta)$ that now appears in the first term of the left-hand side of (27). It is also trivially true that $D > r_o - (1+\theta)^{-1} - \lambda\nu$. Even though ω has now a different meaning, the same analysis of the previous section for optimal ω^* can be entirely applied. Hence, $\partial\omega^*/\partial\varepsilon < 0$ and $\partial\omega^*/\partial\lambda < 0$ as before.

To obtain the stochastic process that generates wealth, we substitute the optimal $\omega^* \in (0,1)$ and c_t^* into (25), finding that (20) still holds. Hence, consumption is given by

$$c_t^* = \frac{\theta r_o}{1+\theta} W_0 e^{\delta_t(\omega^*)}. \tag{28}$$

We also find in this extension that assuming that the investor's time-preference rate equals the world's interest rate does not ensure a steady-state level of consumption. Finally, it is important to point out that, in the stochastic framework, Pareto-Edgeworth independence does not lead to exchange-rate policy neutrality since c_t^* depends on W_t, which, in turn, depends explicitly on ε and λ.

6. Summary and Conclusions

We have used a stylized model of a small open stochastic economy to analyze the response of investment to political risk. The paper provides an explanation of the sharp fall of investment, as occurred in the Mexican case of 1992-1994. The broad message of this paper, although only demonstrated for a specific form of utility index, is that if the investors were uncertain of any government commitment to defend the exchange-rate policy, the anticipated temporariness of the plan to stabilize the exchange rate might lead to an increase in consumption, which in turn causes a reduction on investment.

It is important to point out that the obtained results depend strongly on the assumption of logarithmic utility, which is a limit case of the family of constant relative risk aversion utility functions. The extension of our stochastic analysis to such a family it is not straightforward because of technical difficulties introduced by the jump component in the exchange-rate process. In such a case, results might only be obtained via numerical methods. Needless to say, further work is needed in the above aspect.

Appendix I

The Hamilton-Jacobi-Bellman equation for the stochastic optimal control problem of maximizing (13) with $\log(c_t) = \log(\alpha^{-1} W_t \omega_t)$ and subject to (11) is

$$\begin{aligned}\max_{\omega} H(\omega_t; W_t, t) \equiv \max_{\omega} \Big\{ & \log(\alpha^{-1} W_t \omega_t) e^{-r_o t} + I_W(W_t, t) W_t (r_o - \rho\omega_t) + I_t(W_t, t) \\ & + \frac{1}{2} I_{WW}(W_t, t) W_t^2 \left[\omega_t^2 \sigma^2 + (1-\omega_t)^2 \sigma_0^2 - 2\omega_t (1-\omega_t) \operatorname{Cov}(\mathrm{d}x_t, \mathrm{d}z_t) \right] \\ & + \lambda \left[I\left(W_t \left(\frac{1 + \nu(1-\omega_t)}{1+\nu} \right), t \right) - I(W_t, t) \right] \Big\} = 0, \end{aligned} \tag{I.1}$$

where

$$I(W_t, t) = \max_{\omega} \; \mathrm{E}_t \int_t^{\infty} \log(\alpha^{-1} W_t \omega_t) e^{-r_o s} \mathrm{d}s$$

is the agent's indirect utility function and $I_W(W_t, t)$ is the co-state variable. The first-order condition for an interior solution is:

$$H_{\omega} = 0. \tag{I.2}$$

Given the exponential time discounting in (13), we postulate $I(W_t, t)$ in a time-separable form as

$$I(W_t, t) = e^{-r_o t}\left[\delta_1 \log(W_t) + \delta_o\right], \tag{I.3}$$

where δ_o and δ_1 are to be determined from (I.1). Substituting (I.3) into (I.1), and then computing the first-order conditions in (I.2), we find that $\omega_t \equiv \omega$ is time invariant and

$$\frac{1}{\delta_1 \omega} - \frac{\lambda \nu}{1 + \nu(1 - \omega)} = A + \omega B. \tag{I.4}$$

Coefficients δ_o and δ_1 are determined from (I.1). Thus, $\delta_1 = r_o^{-1}$ and

$$\delta_o = \frac{1}{r_o}\left[1 + \log(\alpha^{-1}\omega^*)\right] - \frac{1}{r_o^2}\left[A\omega^* + \frac{1}{2}\left[(\omega^*)^2 B + \sigma_o^2\right] - \lambda \log\left(\frac{1 + \nu(1 - \omega^*)}{1 + \nu}\right)\right].$$

Equation (I.4) is cubic with one negative and two positive roots, and only one root satisfying $0 < \omega^* < 1$, as sketched in Figure 1. There is also a transversality condition,

$$\lim_{t \to \infty} I(W_t, t) = 0,$$

that is satisfied.

Appendix II

The Hamilton-Jacobi-Bellman equation for the stochastic optimal control problem of maximizing (26) subject to (25) is

$$\max_{c,\omega} H(c_t,\omega_t;W_t,t) \equiv \max_{c,\omega}\Big\{\theta\big[\log(c_t)+\log(W_t\omega_t)\big]e^{-r_o t}$$
$$+ I_W(W_t,t)\,W_t(r_o-\phi\omega_t) - I_W(W_t,t)c_t + I_t(W_t,t)$$
$$+\frac{1}{2}I_{WW}(W_t,t)W_t^2\big[\omega_t^2\sigma^2+(1-\omega_t)^2\sigma_o^2-2\omega_t(1-\omega_t)\mathrm{Cov}(\mathrm{d}x_t,\mathrm{d}z_t)\big]$$
$$+\lambda\left[I\left(W_t\left(\frac{1+\nu(1-\omega_t)}{1+\nu}\right),t\right)-I(W_t,t)\right]\Big\}=0. \tag{II.1}$$

The first-order conditions for an interior solution are given by:

$$H_c = 0 \quad \text{and} \quad H_\omega = 0. \tag{II.2}$$

We postulate $I(W_t,t)=e^{-r_o t}\big[\delta_1\log(W_t)+\delta_o\big]$, where δ_o and δ_1 are to be determined from (II.l). Substituting I (W_t, t) into (II.l), and then computing the first-order conditions in (II.2), we find that $\omega_t \equiv \omega$ is time invariant,

$$c_t=\frac{\theta W_t}{\delta_1} \quad \text{and} \quad \frac{1}{\delta_1\omega}-\frac{\lambda\nu}{1+\nu(1-\omega)}=D+\omega B, \tag{II.3}$$

Coefficients δ_o and δ_1 are determined from (II.1). Thus, $\delta_1=(1+\theta)/r_o$, and

$$\delta_o=\frac{1}{r_o}\left[1+\theta\log\left(\frac{\theta r_o}{1+\theta}\right)+\log(\omega^*)\right]$$
$$-\frac{1+\theta}{r_o^2}\left[D\omega^*+\frac{1}{2}\left[(\omega^*)^2B+\sigma_o^2\right]-\lambda\log\left(\frac{1+\nu(1-\omega^*)}{1+\nu}\right)\right]. \tag{II.4}$$

The second equation in (II.3) has the same properties as those in (I.4).

Appendix III

The generalized Itô's lemma for mixed diffusion-jump processes can be enunciated as follows (see, for instance, Gihman and Skorohod, 1972, chapter 3): Given the homogeneous linear stochastic differential equation

$$\mathrm{d}y_t = y_t\left(\mu \mathrm{d}t + \sigma_1 \mathrm{d}z_{1t} + \sigma_2 \mathrm{d}z_{2t} + \theta \mathrm{d}q_t\right)$$

and $g(y_t)$ twice continuously differentiable, then the "stochastic" differential of $g(y_t)$ is given by

$$\mathrm{d}g(y_t) = \left\{ g_y(y_t)\mu y_t + \frac{1}{2} g_{yy}(y_t)\left[\sigma_1^2 + \sigma_2^2 + 2\mathrm{Cov}(\mathrm{d}z_{1t}, \mathrm{d}z_{2t})\right] y_t^2 \right\}\mathrm{d}t$$
$$+ g_y(y_t)\left[\sigma_1 \mathrm{d}z_{1t} + \sigma_2 \mathrm{d}z_{2t}\right] y_t + \left[g(y_t)(1+\theta) - g(y_t)\right]\mathrm{d}q_t. \qquad \text{(III.1)}$$

Equation (9) follows from a simple application of (III.l). The solution to the homogeneous linear stochastic differential equation

$$\mathrm{d}y_t = y_t\left(\mu \mathrm{d}t + \sigma_1 \mathrm{d}z_{1t} + \sigma_2 \mathrm{d}z_{2t} + \theta \mathrm{d}q_t\right)$$

is given by

$$y_t = y_o \exp\left\{ \left[\mu - \frac{1}{2}\left(\sigma_1^2 + \sigma_2^2 + 2\mathrm{Cov}(\mathrm{d}z_{1t}, \mathrm{d}z_{2t})\right) \right] t \right.$$
$$\left. + \sigma_1 \int_0^t \mathrm{d}z_{1u} + \sigma_2 \int_0^t \mathrm{d}z_{2u} + \log(1+\theta) \int_0^t \mathrm{d}q_u \right\} \qquad \text{(III.2)}$$

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